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Null half-supersymmetric solutions in five-dimensional supergravity

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Abstract: We classify half-supersymmetric solutions of gauged $N = 2$, $D = 5$ supergravity coupled to an arbitrary number of abelian vector multiplets for which all of the Killing spinors generate null Killing vectors. We show that there are four classes of solutions, and in each class we find the metric, scalars and gauge field strengths. When the scalar manifold is symmetric, the solutions correspond to a class of local near horizon geometries recently found by Kunduri and Lucietti.

Keywords: Supergravity Models, Black Holes in String Theory.

JHEP10(2008)103

Contents

1. Introduction

The classification of supersymmetric supergravity solutions has many applications in string theory. The first classification of a five-dimensional supergravity theory was undertaken in [1], in which all supersymmetric solutions of minimal, ungauged $N = 2$, $N = 5$ supergravity were classified. It was constructed by examining the agebraic and differential constraints imposed on spacetime forms obtained from bilinears in the Killing spinors. Such classifications have been used to construct new black hole and black ring solutions. Furthermore, more recent classifications using spinorial geometry techniques can also be

JHEP10(2008)103

used to prove non-existence theorems in several supergravity theories, whereby solutions preserving certain proportions of supersymmetry are excluded. Recently, a partial classification of solutions of gauged $N = 2$, $D = 5$ supergravity was constructed [2]. Solutions with four linearly independent Killing spinors for which at least two generate a timelike Killing vector were completely classified. In this paper we complete the classification of half-supersymmetric solutions of gauged $N = 2$, $D = 5$ supergravity by considering the case when all four Killing spinors generate null Killing vectors.

There are a number of interesting supersymmetric solutions in $N = 2$, $D = 5$ supergravity. Supersymmetric solutions can in principle preserve 1/4, 1/2, 3/4 or the maximal proportion of supersymmetry. Examples of 1/4 supersymmetric solutions are for instance the regular asymptotically AdS_5 black holes found in [3] and later generalized in [4] and [5]. 1/4-supersymmetric string solutions have also been constructed in [6] and [7]. In [8] a classification of all 1/4-supersymmetric solutions of minimal gauged $N = 2, D = 5$ supergravity was performed, this was later extended to a classification of 1/4-supersymmetric solutions of a more general $N = 2$, $D = 5$ gauged supergravity, coupled to an arbitrary number of abelian vector multiplets. Examples of 1/2-supersymmetric solutions are the domain wall solutions in [10], as well as the solutions given in [11, 12] and [13] which correspond to black holes without regular horizons. The regular asymptotically AdS_5 black holes also undergo supersymmetry enhancement in their near-horizon limit from 1/4 to 1/2 supersymmetry, as do the black string solutions in [7]. In [14], it was shown that all 3/4-supersymmetric solutions must be locally AdS_5, although globally there exist discrete qoutients of AdS_5 which are 3/4-supersymmetric [15]. The unique maximally supersymmetric solution is AdS_5.

In order to investigate half-supersymmetric null solutions we will make use of the spinorial geometry method. This method was first used to classify solutions of supergravity theories in ten and eleven dimensions [16, 17]. The first step of such analysis is to write the spinors of the theory as differential forms. The gauge symmetries of the supergravity theories are then used to simplify the spinors as much as possible. By choosing an appropriate basis, the Killing spinor equations (or their integrability conditions) are written as a linear system. This linear system can be solved to express the fluxes of the theory in terms of the geometry and to find the conditions on the geometry imposed by supersymmetry. These methods have also been particularly useful in classifying solutions which preserve very large amounts of supersymmetry; for example in [18] it has been shown that all solutions preserving 29/32, 30/32 or 31/32 of the supersymmetry are in fact maximally supersymmetric. We also remark that the spinorial geometry method has been used to classify solutions of $N = 2$, $D = 4$ supergravity; see for example [19].

The plan of this paper is as follows. In section 2, we review some of the properties of five dimensional gauged supergravity coupled to abelian vector multiplets. In section 3, we show how spinors of the theory can be written as differential forms, and introduce an adapted basis in the forms suitable for defining null Killing spinors. We then use the $\mathrm{Spin}(4,1)$ gauge freedom present in the theory to reduce one null Killing spinor into a particularly simple canonical form, and the residual symmetry present to place the other null spinor into one of two forms. In section 4, we summarize the constraints imposed by solutions preserving 1/4 of the supersymmetry. In sections 5 and 6 we derive constraints

JHEP10(2008)103

on the spacetime geometry, the gauge field strengths and the scalars obtained from the Killing spinor equations. A number of different cases are examined in detail, corresponding to the various different ways in which the Killing spinors can be simplified using gauge transformations. In section 7, we present a self-contained summary of the metrics, scalars and gauge field strengths for all of these half-supersymmetric solutions, together with an interpretation of these solutions. Finally, in appendix A, we show that the integrability conditions of the Killing spinor equations together with the Bianchi identity are sufficient to ensure that the Einstein, gauge and scalar equations hold automatically. In appendix B, we present a detailed derivation of the linear system obtained from the Killing spinor equations for half-supersymmetric null solutions.

2. $N = 2, D = 5$ supergravity

We begin by briefly reviewing some aspects of $N = 2$, $D = 5$ gauged supergravity coupled to n abelian vector multiplets. The bosonic action of this theory is [20]

$$S = \frac{1}{16\pi G}\int \Big((-{}^5R + 2\chi^2\mathcal{V}) \star 1 - Q_{IJ}F^I \wedge *F^J + Q_{IJ}dX^I \wedge \star dX^J \\ -\frac{1}{6}C_{IJK}F^I \wedge F^J \wedge A^K \Big) \tag{2.1}$$

where I, J, K take values $1, \ldots, n$ and $F^I = dA^I$. C_{IJK} are constants that are symmetric on IJK; we will assume that Q_{IJ} is invertible, with inverse Q^{IJ}. The metric has signature $(+, -, -, -, -)$.

The X^I are scalars which are constrained via

$$\frac{1}{6}C_{IJK}X^I X^J X^K = 1\,. \tag{2.2}$$

We may regard the X^I as being functions of $n-1$ unconstrained scalars ϕ^a. It is convenient to define

$$X_I \equiv \frac{1}{6}C_{IJK}X^J X^K \tag{2.3}$$

so that the condition (2.2) becomes

$$X_I X^I = 1\,. \tag{2.4}$$

In addition, the coupling Q_{IJ} depends on the scalars via

$$Q_{IJ} = \frac{9}{2}X_I X_J - \frac{1}{2}C_{IJK}X^K \tag{2.5}$$

so in particular

$$Q_{IJ}X^J = \frac{3}{2}X_I\,, \qquad Q_{IJ}\partial_a X^J = -\frac{3}{2}\partial_a X_I\,. \tag{2.6}$$

The scalar potential can be written as,

$$\mathcal{V} = 9V_I V_J\Big(X^I X^J - \frac{1}{2}Q^{IJ}\Big)\,, \tag{2.7}$$

where V_I are constants.

For a bosonic background to be supersymmetric there must be a spinor ϵ for which the supersymmetry variations of the gravitino and the superpartners of the scalars vanish. We shall investigate the properties of these spinors in greater detail in the next section. The gravitino Killing spinor equation is

$$\left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{\rho\sigma}\Gamma_{\rho\sigma} - \frac{3i\chi}{2}A_\mu + \frac{i\chi}{2}V_I X^I\Gamma_\mu - \frac{3}{4}H_\mu{}^\rho\Gamma_\rho + \frac{1}{8}\Gamma_\mu H^{\rho\sigma}\Gamma_{\rho\sigma}\right)\epsilon = 0\,, \tag{2.8}$$

where ϵ is a Dirac spinor. The algebraic Killing spinor equations associated with the variation of the scalar superpartners are

$$\left(4i\chi\left(X^I V_J X^J - \frac{3}{2}Q^{IJ}V_J\right) + 2\partial^\mu X^I\Gamma_\mu - (F^{I\mu\nu} - X^I H^{\mu\nu})\Gamma_{\mu\nu}\right)\epsilon = 0\ . \tag{2.9}$$

where we define $H = X_I F^I, A = V_I A^I$. We shall refer to (2.9) as the dilatino Killing spinor equation. We also require that the bosonic background should satisfy the Einstein, gauge field and scalar field equations obtained from the action (2.1) and analyse these in appendix A.

3. Spinors in five dimensions

Following [21 – 23], the space of Dirac spinors in five dimensions is the space of complexified forms on $\mathbb{R}^2$, $\Delta = \Lambda^*(\mathbb{R}^2)\otimes\mathbb{C}$. A generic spinor η can therefore be written as

$$\eta = \lambda 1 + \mu^i e^i + \sigma e^{12}\,, \tag{3.1}$$

where e^1, e^2 are 1-forms on $\mathbb{R}^2$, and $i = 1, 2$ for complex functions λ, μ^i and σ. The action of γ-matrices on these forms is given by

$$\gamma_i = i(e^i \wedge + i_{e^i})\,, \tag{3.2}$$

$$\gamma_{i+2} = -e^i \wedge + i_{e^i}\,, \tag{3.3}$$

for $i = 1, 2$. γ_0 is defined by

$$\gamma_0 = \gamma_{1234}\,, \tag{3.4}$$

and satisfies

$$\gamma_0 1 = 1, \quad \gamma_0 e^{12} = e^{12}, \quad \gamma_0 e^i = -e^i \quad i = 1, 2\ . \tag{3.5}$$

The charge conjugation operator C is defined by

$$C1 = -e^{12}, \quad Ce^{12} = 1 \quad Ce^i = -\epsilon_{ij}e^j \quad i = 1, 2 \tag{3.6}$$

where $\epsilon_{ij} = \epsilon^{ij}$ is antisymmetric with $\epsilon_{12} = 1$. We also note the useful identity

$$(\gamma_M)^* = -\gamma_0 C\gamma_M\gamma_0 C\ . \tag{3.7}$$

3.1 A Spin(4, 1) invariant bilinear form on spinors

In order to analyse the 1/2 supersymmetric solutions it will be useful to construct a non-degenerate bilinear form on the space of spinors. To do so, we first define a Hermitian inner product on the space of spinors via

$$\langle z^0.1 + z^1 e^1 + z^2 e^2 + z^3 e^{12}, w^0.1 + w^1 e^1 + w^2 e^2 + w^3 e^{12} \rangle = \bar{z}^\alpha w^\alpha \,, \tag{3.8}$$

summing over $\alpha = 0, 1, 2, 3$. However, $\langle , \rangle$ is not Spin(4, 1) gauge-invariant. To rectify this, we define an inner product B given by

$$B(\eta, \epsilon) = \langle C\eta^*, \epsilon \rangle \,, \tag{3.9}$$

which satisfies the identities

$$\begin{aligned} B(\eta, \epsilon) + B(\epsilon, \eta) &= 0 \,, \\ B(\gamma_M \eta, \epsilon) - B(\eta, \gamma_M \epsilon) &= 0 \,, \\ B(\gamma_{MN} \eta, \epsilon) + B(\eta, \gamma_{MN} \epsilon) &= 0 \,, \end{aligned} \tag{3.10}$$

for all spinors η, ϵ.

In particular, the last of the above constraints implies that B is Spin(4, 1) invariant. Note that B is linear over $\mathbb{C}$ in both arguments. B is also non-degenerate: if $B(\epsilon, \eta) = 0$ for all η then $\epsilon = 0$.

3.2 The null basis

To work in a basis adapted to describing solutions with Killing spinors which generate null Killing vectors, define

$$\begin{aligned} \Gamma_\pm &= \frac{1}{\sqrt{2}}(\gamma_0 \mp \gamma_3) \,, \\ \Gamma_1 &= \frac{1}{\sqrt{2}}(\gamma_2 - i\gamma_4) = \sqrt{2} i e^2 \wedge \,, \\ \Gamma_{\bar{1}} &= \frac{1}{\sqrt{2}}(\gamma_2 + i\gamma_4) = \sqrt{2} i i_{e^2} \,, \\ \Gamma_2 &= \gamma_1 \,. \end{aligned} \tag{3.11}$$

We then define a basis for the Dirac spinors Δ by

$$\psi^1_\pm = 1 \pm e^1, \qquad \psi^{\bar{1}}_\pm = e^{12} \mp e^2 \,. \tag{3.12}$$

Note that $\psi^1_\pm$ is *not* the complex conjugate of $\psi^{\bar{1}}_\pm$.

Then it is straightforward to show that

$$\begin{aligned} \Gamma_\pm \psi^\alpha_\pm &= 0 \\ \Gamma_\pm \psi^\alpha_\mp &= \sqrt{2} \psi^\alpha_\pm \\ \Gamma_1 \psi^1_\pm &= \mp \sqrt{2} i \psi^{\bar{1}}_\pm \\ \Gamma_{\bar{1}} \psi^{\bar{1}}_\pm &= \mp \sqrt{2} i \psi^1_\pm \end{aligned}$$

$$
\begin{aligned}
\Gamma_1\psi^{\bar{1}}_{\pm} &= 0\\
\Gamma_{\bar{1}}\psi^{1}_{\pm} &= 0\\
\Gamma_2\psi^{1}_{\pm} &= \pm i\psi^{1}_{\pm}\\
\Gamma_2\psi^{\bar{1}}_{\pm} &= \mp i\psi^{\bar{1}}_{\pm}\,,
\end{aligned}
\tag{3.13}
$$

where $\alpha, \beta = 1, \bar{1}$.

A generic spinor can then be written as

$$
\eta = \lambda^{\alpha}_{+}\psi^{\alpha}_{+} + \lambda^{\alpha}_{-}\psi^{\alpha}_{-}\,,
\tag{3.14}
$$

where there is summation over $\alpha = 1, \bar{1}$. Note that the $\lambda^{\alpha}_{\pm}$ are in general complex and $\lambda^{1}_{\pm}$ is *not* the complex conjugate of $\lambda^{\bar{1}}_{\pm}$.

The metric has vielbein $\mathbf{e}^{+}, \mathbf{e}^{-}, \mathbf{e}^{1}, \mathbf{e}^{\bar{1}}, \mathbf{e}^{2}$, where $\mathbf{e}^{\pm}, \mathbf{e}^{2}$ are real, and $\mathbf{e}^{1}, \mathbf{e}^{\bar{1}}$ are complex conjugate, and

$$
ds^2 = 2\mathbf{e}^{+}\mathbf{e}^{-} - 2\mathbf{e}^{1}\mathbf{e}^{\bar{1}} - (\mathbf{e}^{2})^2\ .
\tag{3.15}
$$

Now note that on writing the Dirac spinor η as $\eta = \eta^1 + i\eta^2$, where η^a are symplectic Majorana spinors, we find

$$
B(\eta^1, \eta^2) = \frac{1}{2}B(\gamma_0 C\eta^*, \eta) = -\frac{1}{2}\langle\gamma_0\eta, \eta\rangle\ .
\tag{3.16}
$$

Hence the nullity condition $B(\eta^1, \eta^2) = 0$ can be rewritten in the null basis as

$$
\lambda^{1}_{+}(\lambda^{1}_{-})^* + (\lambda^{1}_{+})^*\lambda^{1}_{-} + \lambda^{\bar{1}}_{+}(\lambda^{\bar{1}}_{-})^* + (\lambda^{\bar{1}}_{+})^*\lambda^{\bar{1}}_{-} = 0\ .
\tag{3.17}
$$

To proceed further, note that

$$
e^{x\gamma_{03}+y\gamma_{24}}(1 + e^1) = e^{x-iy}(1 + e^1)\,,
\tag{3.18}
$$

for $x, y \in \mathbb{R}$, and it is also convenient to define

$$
T_1 = \gamma_{01} + \gamma_{13}, \quad T_2 = \gamma_{02} + \gamma_{23}, \quad T_3 = \gamma_{04} - \gamma_{34}\,,
\tag{3.19}
$$

which satisfy

$$
T_i\psi^{\alpha}_{+} = 0\,,
\tag{3.20}
$$

for $\alpha = 1, \bar{1}$, and also

$$
T_1\psi^{1}_{-} = -2i\psi^{1}_{+}, \qquad T_1\psi^{\bar{1}}_{-} = 2i\psi^{\bar{1}}_{+}\ ,
\tag{3.21}
$$

$$
T_2\psi^{1}_{-} = 2i\psi^{\bar{1}}_{+}, \qquad T_2\psi^{\bar{1}}_{-} = 2i\psi^{1}_{+}\,,
\tag{3.22}
$$

$$
T_3\psi^{1}_{-} = -2\psi^{\bar{1}}_{+}, \qquad T_3\psi^{\bar{1}}_{-} = 2\psi^{1}_{+}\ .
\tag{3.23}
$$

Note that gauge transformations of the form $e^{xT_1+yT_2+zT_3}$ for $x, y, z \in \mathbb{R}$ map

$$
\begin{aligned}
\lambda^{\alpha}_{-} &\to \lambda^{\alpha}_{-}\\
\lambda^{1}_{+} &\to \lambda^{1}_{+} - ix\lambda^{1}_{-} + (z + iy)\lambda^{\bar{1}}_{-}\\
\lambda^{\bar{1}}_{+} &\to \lambda^{\bar{1}}_{+} + ix\lambda^{\bar{1}}_{-} + (iy - z)\lambda^{1}_{-}\ .
\end{aligned}
\tag{3.24}
$$

Clearly these leave $1+e^1$ invariant. We therefore adopt the following approach. Using the Spin$(4,1)$ gauge freedom, we can choose without loss of generality the first Killing spinor to be

$$\epsilon = \psi^1_+ \ . \tag{3.25}$$

The gauge transformations $e^{xT_1+yT_2+zT_3}$ leave ϵ invariant. The second Killing spinor of the form

$$\eta = \lambda^\alpha_+\psi^\alpha_+ + \lambda^\alpha_-\psi^\alpha_- \tag{3.26}$$

where $\lambda^\alpha_\pm$ satisfy (3.17) can then be simplified by using the gauge transformations $e^{xT_1+yT_2+zT_3}$.

In particular, we note that we can make use of the gauge transformations to set either $\lambda^\alpha_+ = 0$, or $\lambda^\alpha_- = 0$. To see this, let us first assume that $\lambda^1_- \neq 0$ and $\lambda^{\bar{1}}_- \neq 0$. Then we can use (3.24) to set $\lambda^1_+ = 0$ by imposing

$$(z+iy)\lambda^{\bar{1}}_- - ix\lambda^1_- = \lambda^1_+ \ . \tag{3.27}$$

This fixes z, y in the $\lambda^{\bar{1}}_+$ transformation

$$\begin{aligned}\lambda^{\bar{1}}_+ &\to \lambda^{\bar{1}}_+ + ix\lambda^{\bar{1}}_- - \frac{\lambda^1_-}{\lambda^{\bar{1}\,*}_-}\big(\lambda^{1\,*}_+ - ix\lambda^{1\,*}_-\big) \\ &= \frac{1}{\lambda^{\bar{1}\,*}_-}\big(\lambda^{\bar{1}}_+\lambda^{\bar{1}\,*}_- - \lambda^1_-\lambda^{1\,*}_+ + ix(\lambda^{\bar{1}}_-\lambda^{\bar{1}\,*}_- + \lambda^1_-\lambda^{1\,*}_-)\big) \ .\end{aligned} \tag{3.28}$$

We can fix x here such that the term in brackets is real; then we find

$$\begin{aligned}\lambda^1_+ &= 0 \\ \lambda^{\bar{1}}_+ &= \mu\lambda^{\bar{1}}_- \,,\end{aligned} \tag{3.29}$$

with $\mu \in \mathbb{R}$. To proceed further we use this result together with the nullity condition (3.17) to find

$$2\mu\lambda^{\bar{1}}_-\lambda^{\bar{1}\,*}_- = 0 \ . \tag{3.30}$$

This implies that $\mu = 0$. Alternatively, we have the case where $\lambda^1_- = 0, \lambda^{\bar{1}}_- \neq 0$. Here we can use y, z in (3.24) to set $\lambda^1_+ = 0$. This sets

$$\begin{aligned}\lambda^{\bar{1}}_+ &\to \lambda^{\bar{1}}_+ + ix\lambda^{\bar{1}}_- \\ &= \lambda^{\bar{1}}_-\left(ix + \frac{\lambda^{\bar{1}}_+}{\lambda^{\bar{1}}_-}\right) \ .\end{aligned} \tag{3.31}$$

Here x can be chosen to set the term in brackets to be real, so that once again we have

$$\begin{aligned}\lambda^1_+ &= 0 \\ \lambda^{\bar{1}}_+ &= \mu\lambda^{\bar{1}}_- = 0 \,,\end{aligned} \tag{3.32}$$

where we set $\mu = 0$ using the nullity condition as before. The case $\lambda^1_- \neq 0, \lambda^{\bar{1}}_- = 0$ proceeds analogously.

4. Quarter-supersymmetric null solutions

In appendix B we arrive at the general linear system following from the dilatino and gravitino equations acting on a spinor $\epsilon = \lambda^1_+\psi^1_+ + \lambda^{\bar{1}}_+\psi^{\bar{1}}_+ + \lambda^1_-\psi^1_- + \lambda^{\bar{1}}_-\psi^{\bar{1}}_-$. Restricting to the case $\epsilon = \psi^1_+$ we find

$$F^I_{+-} = 0\,, \tag{4.1}$$

$$F^I_{1\bar{1}} = -i\Big(-\partial_2 X^I + 2\chi\Big(X^I V_J X^J - \frac{3}{2}Q^{IJ}V_J\Big)\Big) + X^I H_{1\bar{1}}\,, \tag{4.2}$$

$$\partial_+ X^I = 0\,, \tag{4.3}$$

$$F^I_{+2} = 0\,, \tag{4.4}$$

$$F^I_{+\bar{1}} = 0\,, \tag{4.5}$$

$$F^I_{\bar{1}2} = i\partial_{\bar{1}}X^I + X^I H_{\bar{1}2}\ . \tag{4.6}$$

Further constraints on the spin connection obtained from the gravitino equation acting on $\epsilon = \psi^1_+$ are

$$\omega_{+,+-} = \omega_{+,+2} = \omega_{+,+\bar{1}} = \omega_{-,+-} = \omega_{1,+2} = \omega_{\bar{1},+\bar{1}} = 0, \tag{4.7}$$

and

$$\omega_{\bar{1},\bar{1}2} = \omega_{2,+2} = \omega_{+,\bar{1}2} = \omega_{2,+1} = \omega_{1,+\bar{1}} = 0\,, \tag{4.8}$$

as well as

$$\omega_{\bar{1},+-} + \omega_{-,+\bar{1}} = 0\,, \tag{4.9}$$

$$-\omega_{\bar{1},+-} + \frac{1}{2}\omega_{2,\bar{1}2} = 0\,, \tag{4.10}$$

$$-2i\omega_{-,+2} + i\omega_{1,\bar{1}2} + 3i\chi V_I X^I = 0\,, \tag{4.11}$$

$$\omega_{2,+-} + \omega_{-,+2} = 0\,, \tag{4.12}$$

We also find

$$H_{+1} = H_{+2} = H_{+-} = 0\,, \tag{4.13}$$

$$H_{-\bar{1}} = -\frac{2i}{3}\omega_{-,\bar{1}2}\,, \tag{4.14}$$

$$H_{-2} = 2\chi A_- - \frac{2i}{3}\omega_{-,1\bar{1}}\,, \tag{4.15}$$

$$H_{\bar{1}2} = 2i\omega_{\bar{1},+-}\,, \tag{4.16}$$

$$H_{1\bar{1}} = -\frac{2i}{3}\omega_{-,+2} - \frac{2i}{3}\omega_{1,\bar{1}2}\,, \tag{4.17}$$

where the gauge potential has the following components constrained

$$\chi A_{\bar{1}} = \frac{2i}{3}\omega_{\bar{1},+-} + \frac{i}{3}\omega_{\bar{1},1\bar{1}}\,, \tag{4.18}$$

$$\chi A_2 = \frac{i}{3}\omega_{2,1\bar{1}}\,, \tag{4.19}$$

$$\chi A_+ = \frac{i}{3}\omega_{+,1\bar{1}}\ . \tag{4.20}$$

To proceed to half-supersymmetric solutions, we incorporate these constraints into the full linear system in appendix B and consider two cases in which either $\lambda^\alpha_+ = 0$ or $\lambda^\alpha_- = 0$.

JHEP10(2008)103

5. Solutions with $\lambda^{\alpha}_{+} = 0$

For this class of solutions, we set $\lambda^{\alpha}_{+} = 0$ for $\alpha = 1, \bar{1}$, in the components of the dilatino and gravitino Killing spinor equations, with the resulting linear system presented in appendix B. For a non-trivial solution to (B.69), and (B.70) to exist, we require

$$2\Big(-\chi A_- - \frac{i}{3}\omega_{-1\bar{1}} - \frac{i}{2}\omega_{2,-2}\Big)\Big(-\chi A_- - \frac{i}{3}\omega_{-,1\bar{1}} - \frac{i}{2}\omega_{2,-2}\Big) + \Big(-\omega_{2,-\bar{1}} + \frac{1}{3}\omega_{-,\bar{1}2}\Big)\Big(-\omega_{2,-1} + \frac{1}{3}\omega_{-,12}\Big) = 0\,, \tag{5.1}$$

which implies that

$$\chi A_- = \frac{i}{3}\omega_{-,1\bar{1}}\,, \tag{5.2}$$
$$\omega_{2,-2} = 0\,, \tag{5.3}$$
$$\omega_{2,-\bar{1}} = \frac{1}{3}\omega_{-,\bar{1}2}\ . \tag{5.4}$$

Using (B.61), and (B.66) we require

$$\frac{1}{2}(\omega_{-,12} + \omega_{1,-2})(\omega_{-,\bar{1}2} + \omega_{\bar{1},-2}) + (\omega_{1,-1})(\omega_{\bar{1},-\bar{1}}) = 0\,, \tag{5.5}$$

which implies that

$$\omega_{-,12} = -\omega_{1,-2}\,, \tag{5.6}$$
$$\omega_{1,-1} = 0\ . \tag{5.7}$$

We can also use (B.57), and (B.58) finding that

$$(\omega_{-,-2})(\omega_{-,-2}) + (\omega_{-,-\bar{1}})(\omega_{-,-1}) = 0\,, \tag{5.8}$$

so that

$$\omega_{-,-2} = 0\,, \tag{5.9}$$
$$\omega_{-,-1} = 0\ . \tag{5.10}$$

From (B.62), and (B.65)

$$(\omega_{1,-\bar{1}})(\omega_{\bar{1},-1}) + \frac{8}{9}(\omega_{1,-2})(\omega_{\bar{1},-2}) = 0\,, \tag{5.11}$$

from which we see that

$$\omega_{1,-\bar{1}} = 0\,, \tag{5.12}$$
$$\omega_{1,-2} = \omega_{-,12} = \omega_{2,-1} = 0\ . \tag{5.13}$$

Using the dilatino equations (B.49) and (B.50), we require that

$$8\big(\partial_- X^I - i(F^I_{-2} - X^I H_{-2})\big)\big(\partial_- X^J + i(F^J_{-2} - X^J H_{-2})\big) + 16(F^I_{-\bar{1}} - X^I H_{-\bar{1}})(F^J_{-1} - X^J H_{-1}) = 0\,, \tag{5.14}$$

so that, upon contracting with Q_{IJ}

$$F^I_{-\bar{1}} = X^I H_{-\bar{1}} = 0\,, \tag{5.15}$$
$$F^I_{-2} = X^I H_{-2} = 0\,, \tag{5.16}$$
$$\partial_- X^I = 0\,. \tag{5.17}$$

Within the case $\lambda^\alpha_+ = 0$ there are three sub-cases to consider. Here either ($\lambda^1_- \neq 0, \lambda^{\bar{1}}_- \neq 0$), or ($\lambda^1_- = 0, \lambda^{\bar{1}}_- \neq 0$), or ($\lambda^1_- \neq 0, \lambda^{\bar{1}}_- = 0$). Before analysing these three cases in detail, we compute the stability group $\mathrm{Stab}(\eta_1, \eta_2)$ which leaves the spinors $\eta_1 = \psi^1_+$ and $\eta_2 = \lambda^1_- \psi^1_- + \lambda^{\bar{1}}_- \psi^{\bar{1}}_-$ invariant. We also evaluate the Σ-groups, which were introduced in the context of an analysis of certain ten-dimensional supergravity solutions in [24]. For the solutions under consideration here, the Σ-group is

$$\Sigma(\mathcal{P}) = \mathrm{Stab}(\mathcal{P})/\mathrm{Stab}(\eta_1, \eta_2) \tag{5.18}$$

where $\mathcal{P}$ denotes the 2-dimensional span over $\mathbb{C}$ of η_1, η_2, and $\mathrm{Stab}(\mathcal{P})$ is the subgroup of $\mathrm{Spin}(4,1) \times \mathrm{U}(1)$ which preserves $\mathcal{P}$ (though not η_1 and η_2 individually). In the gauged supergravity theory, there is always a $\mathrm{U}(1)$ factor in $\Sigma(\mathcal{P})$, due to the gauging.

It is straightforward to show that in all three cases,

$$\mathrm{Stab}(\eta_1, \eta_2) = \{1\} \tag{5.19}$$

and for the Σ-groups:

(i) When $\lambda^1_- \neq 0, \lambda^{\bar{1}}_- \neq 0$ the Σ-group is $\mathrm{Spin}(2,1) \times \mathrm{U}(1)$, generated by $\{\Gamma_{+-}, \Gamma_{+2} - \frac{1}{\sqrt{2}}\frac{\lambda^{\bar{1}}_-}{\lambda^1_-}\Gamma_{+1} - \frac{1}{\sqrt{2}}(\frac{\lambda^{\bar{1}}_-}{\lambda^1_-})^*\Gamma_{+\bar{1}}, \Gamma_{-2} - \frac{1}{\sqrt{2}}\frac{\lambda^{\bar{1}}_-}{\lambda^1_-}\Gamma_{-1} - \frac{1}{\sqrt{2}}(\frac{\lambda^{\bar{1}}_-}{\lambda^1_-})^*\Gamma_{-\bar{1}}\}$.

(ii) When $\lambda^1_- = 0, \lambda^{\bar{1}}_- \neq 0$ the Σ-group is $\mathrm{Spin}(3,1) \times \mathrm{U}(1)$, generated by $\{\Gamma_{+-}, \Gamma_{+1}, \Gamma_{+\bar{1}}, \Gamma_{-1}, \Gamma_{-\bar{1}}, \Gamma_{1\bar{1}}\}$.

(iii) When $\lambda^1_- \neq 0, \lambda^{\bar{1}}_- = 0$, the Σ-group is $\mathrm{Spin}(2,1) \times \mathrm{U}(1) \times \mathrm{U}(1)$, generated by $\{\Gamma_{+-}, \Gamma_{+2}, \Gamma_{-2}, \Gamma_{1\bar{1}}\}$.

5.1 Solutions with $\lambda^1_- \neq 0$ and $\lambda^{\bar{1}}_- \neq 0$

Suppose first that $\lambda^1_- \neq 0$ and $\lambda^{\bar{1}}_- \neq 0$. Then note that the $\mathrm{U}(1) \times \mathrm{Spin}(4,1)$ gauge transformation of the type $e^{ig\mu}e^{g\mu\gamma_{24}}$ for $\mu \in \mathbb{R}, g \in \mathbb{R}$ which acts on spinors via

$$\begin{aligned} \psi^1_\pm &\to e^{ig\mu}e^{g\mu\gamma_{24}}\psi^1_\pm = \psi^1_\pm \\ \psi^{\bar{1}}_\pm &\to e^{ig\mu}e^{g\mu\gamma_{24}}\psi^{\bar{1}}_\pm = e^{2ig\mu}\psi^{\bar{1}}_\pm\,, \end{aligned} \tag{5.20}$$

leaves $\epsilon = \psi^1_+$ invariant, and transforms η as

$$\eta \to \lambda^1_-\psi^1_- + e^{2ig\mu}\lambda^{\bar{1}}_-\psi^{\bar{1}}_- = (\lambda^1_-)'\psi^1_- + (\lambda^{\bar{1}}_-)'\psi^{\bar{1}}_-\,. \tag{5.21}$$

Define

$$g = i\log\frac{\lambda^1_-\lambda^{\bar{1}}_-}{(\lambda^1_-\lambda^{\bar{1}}_-)^*}\,. \tag{5.22}$$

Then we find that

$$\frac{(\lambda_-^1)'(\lambda_-^{\bar{1}})'}{((\lambda_-^1)'(\lambda_-^{\bar{1}})')^*} = \left(\frac{\lambda_-^1\lambda_-^{\bar{1}}}{(\lambda_-^1\lambda_-^{\bar{1}})^*}\right)^{1-4\mu} . \tag{5.23}$$

Hence, for $\mu = \frac{1}{4}$, and dropping the primes, we have

$$\frac{\lambda_-^1\lambda_-^{\bar{1}}}{(\lambda_-^1\lambda_-^{\bar{1}})^*} = 1 . \tag{5.24}$$

Now, observe that

$$\partial_+ g = -2i\omega_{+,1\bar{1}}, \qquad \partial_- g = -2i\omega_{-,1\bar{1}} , \tag{5.25}$$

so that, working in this gauge, we can take without loss of generality

$$\omega_{+,1\bar{1}} = \omega_{-,1\bar{1}} = 0 . \tag{5.26}$$

Note in particular that in this gauge

$$\partial_+\lambda_-^{\bar{1}} = \partial_-\lambda_-^{\bar{1}} = \partial_+\lambda_-^1 = \partial_-\lambda_-^1 = 0 . \tag{5.27}$$

To proceed, we investigate several integrability conditions. In particular, requiring that $\nabla_{[+}\nabla_{-]}\lambda_-^{\bar{1}} = 0$ imposes the constraint

$$\begin{aligned}(\omega_{+,-\bar{1}} - \omega_{-,+\bar{1}})(-\omega_{1,1\bar{1}}\lambda_-^{\bar{1}} - \sqrt{2}\omega_{1,\bar{1}2}\lambda_-^1) - (\omega_{+,-1} - \omega_{-,+1})\omega_{\bar{1},1\bar{1}}\lambda_-^{\bar{1}} \\ +(\omega_{+,-2} - \omega_{-,+2})\bigg(- \sqrt{2}\omega_{2,\bar{1}2}\lambda_-^1 + \bigg(- \omega_{2,1\bar{1}} - \frac{1}{3}\omega_{\bar{1},12} + \frac{2}{3}\omega_{-,+2}\bigg)\lambda_-^{\bar{1}}\bigg) = 0 ,\end{aligned} \tag{5.28}$$

and requiring that $\nabla_{[+}\nabla_{-]}\lambda_-^1 = 0$ imposes the constraint

$$\begin{aligned}\bigg(\frac{2\sqrt{2}}{3}(\omega_{+,-1} - \omega_{-,+1})(\omega_{1,\bar{1}2} + \omega_{-,+2}) + \sqrt{2}(\omega_{+,-2} - \omega_{-,+2})\omega_{2,12}\bigg)\lambda_-^{\bar{1}} \\ +2(\omega_{+,-1}\omega_{-,+\bar{1}} - \omega_{+,-\bar{1}}\omega_{-,+\bar{1}})\lambda_-^1 = 0 .\end{aligned} \tag{5.29}$$

Next, the conditions $\nabla_{[\pm}\nabla_{B]}\lambda_-^1 = \nabla_{[\pm}\nabla_{B]}\lambda_-^{\bar{1}} = 0$ for $B = 1, \bar{1}, 2$ impose the constraints

$$\partial_\pm\omega_{2,12} = \partial_\pm\omega_{2,1\bar{1}} = \partial_\pm\omega_{1,+-} = \partial_\pm\omega_{1,1\bar{1}} = \partial_\pm\omega_{1,\bar{1}2} = \partial_\pm\omega_{-,+2} = 0 . \tag{5.30}$$

Now note that in the gauge for which $A_+ = A_- = 0$, we have

$$\chi A = \frac{i}{3}\omega_{2,1\bar{1}}\mathbf{e}^2 + \frac{i}{3}(2\omega_{\bar{1},+-} + \omega_{\bar{1},1\bar{1}})\mathbf{e}^{\bar{1}} - \frac{i}{3}(2\omega_{1,+-} - \omega_{1,1\bar{1}})\mathbf{e}^1 . \tag{5.31}$$

The integrability condition $d(\chi A)_{+-} = 0$ then implies that

$$\begin{aligned}(2\omega_{\bar{1},+-} + \omega_{\bar{1},1\bar{1}})(-\omega_{+,-1} + \omega_{-,+1}) + (-2\omega_{1,+-} + \omega_{1,1\bar{1}})(-\omega_{+,-\bar{1}} + \omega_{-,+\bar{1}}) \\ +\omega_{2,1\bar{1}}(-\omega_{+,-2} + \omega_{-,+2}) = 0 .\end{aligned} \tag{5.32}$$

Note also that (B.53) and (B.54) can be rewritten as

$$\frac{1}{\sqrt{2}}(\omega_{+,-2} + \omega_{-,+2})\lambda_-^1 - (\omega_{+,-1} + \omega_{-,+1})\lambda_-^{\bar{1}} = 0 , \tag{5.33}$$

and

$$-(\omega_{+,-\bar{1}}+\omega_{-,+\bar{1}})\lambda^1_- + \bigg(-\frac{1}{\sqrt{2}}\omega_{+,-2}+\frac{1}{3\sqrt{2}}\omega_{-,+2}-\frac{\sqrt{2}}{3}\omega_{1,\bar{1}2}\bigg)\lambda^{\bar{1}}_- = 0\ . \tag{5.34}$$

Next note that the component of the Bianchi identity $X_I dF^I_{+-2} = 0$ implies that

$$\omega_{-,+1}\omega_{+,-\bar{1}} - \omega_{-,+\bar{1}}\omega_{+,-1} = 0\,, \tag{5.35}$$

and substituting this into (5.29) we find

$$\frac{1}{3}(\omega_{+,-1}-\omega_{-,+1})(\omega_{1,\bar{1}2}-\omega_{-,+2}) - \omega_{-,+1}(\omega_{+,-2}-\omega_{-,+2}) = 0\ . \tag{5.36}$$

Using these identities we obtain the constraints

$$\frac{1}{2}((\omega_{+,-2})^2-(\omega_{-,+2})^2) + \omega_{-,+1}\omega_{-,+\bar{1}} - \omega_{+,-\bar{1}}\omega_{+,-1} = 0\,, \tag{5.37}$$

$$(\omega_{+,-2}+\omega_{-,+2})\bigg((\omega_{+,-\bar{1}}-\omega_{-,+\bar{1}})\lambda^1_- + \frac{1}{\sqrt{2}}(\omega_{+,-2}-\omega_{-,+2})\lambda^{\bar{1}}_-\bigg) = 0\ . \tag{5.38}$$

We now find cases according as to whether $(\omega_{+,-2}+\omega_{-,+2})$ vanishes. First suppose $(\omega_{+,-2}+\omega_{-,+2}) = 0$. Then (5.33) and (5.34) imply that

$$2\omega_{-,+2} - \omega_{1,\bar{1}2} = 3\chi V_I X^I = 0\ . \tag{5.39}$$

Contracting (B.52) with V_I then implies that $Q^{IJ}V_I V_J = 0$. As Q^{IJ} is positive definite this is a contradiction.

We are then led to take $(\omega_{+,-2}+\omega_{-,+2}) \neq 0$. In this case we have the constraint

$$(\omega_{+,-\bar{1}}-\omega_{-,+\bar{1}})\lambda^1_- + \frac{1}{\sqrt{2}}(\omega_{+,-2}-\omega_{-,+2})\lambda^{\bar{1}}_- = 0\ . \tag{5.40}$$

Further simplifications can be made by going back to our gauge transformations (5.20). Requiring $\partial_1 g = 0$ implies that

$$\sqrt{2}\omega_{1,1\bar{1}}\lambda^{\bar{1}}_- = -\omega_{1,\bar{1}2}\lambda^1_-\,, \tag{5.41}$$

when taken together with (5.38). Similarly, $\partial_2 g = 0$ can be shown to require that

$$\chi A_2 = \omega_{2,1\bar{1}} = 0\ . \tag{5.42}$$

These conditions are sufficient to show that

$$d\bigg(\frac{\lambda^{\bar{1}}_-}{(\lambda^{\bar{1}}_-)^*}\bigg) = 0\,, \tag{5.43}$$

and hence from (5.24) that

$$d\bigg(\frac{\lambda^1_-}{(\lambda^1_-)^*}\bigg) = 0\ . \tag{5.44}$$

Then, by making use of the $\mathrm{U}(1) \times \mathrm{Spin}(4,1)$ gauge transformation of the type $e^{i\theta_1} e^{\theta_2 \gamma_{24}}$ for constant $\theta_1, \theta_2 \in \mathbb{R}$, we can set, without loss of generality

$$\frac{\lambda^{\bar{1}}_-}{(\lambda^{\bar{1}}_-)^*} = \frac{\lambda^1_-}{(\lambda^1_-)^*} = 1 \,. \tag{5.45}$$

This gauge transformation multiplies ψ^1_+ by a phase, however as this phase is constant, it does not alter the constraints obtained in the analysis of the quarter-supersymmetric solutions.

Using these results, we find the following constraints remain on the spatial derivatives of the λ's;

$$\partial_1 \lambda^1_- = -2\omega_{-,+1}\lambda^1_- \,, \tag{5.46}$$

$$\partial_1 \lambda^{\bar{1}}_- = \frac{-1}{\sqrt{2}}\omega_{1,\bar{1}2}\lambda^1_- \,, \tag{5.47}$$

$$\partial_2 \lambda^1_- = -2\sqrt{2}\omega_{-,+1}\lambda^{\bar{1}}_- \,, \tag{5.48}$$

$$\partial_2 \lambda^{\bar{1}}_- = -\omega_{1,\bar{1}2}\lambda^{\bar{1}}_- \,. \tag{5.49}$$

To proceed we note that

$$V = \left(\frac{(\lambda^1_-)^2 + (\lambda^{\bar{1}}_-)^2}{\sqrt{2}}\right)\mathbf{e}^+ \,, \tag{5.50}$$

$$W = \mathbf{e}^- \,, \tag{5.51}$$

are Killing vectors of the theory. We can find an additional Killing vector U, as

$$U = [V, W] = c_1 Y \,, \tag{5.52}$$

where Y is defined by

$$Y = \lambda^{\bar{1}}_-(\mathbf{e}^1 + \mathbf{e}^{\bar{1}}) - \sqrt{2}\lambda^1_-\mathbf{e}^2 \,, \tag{5.53}$$

and c_1 by

$$c_1 = \omega_{-,+2}\lambda^1_- - \sqrt{2}\omega_{-,+1}\lambda^{\bar{1}}_- \,. \tag{5.54}$$

As Y can also be shown to be Killing we find that c_1 must be a constant.

We define a vector orthogonal to V, W, and Y as

$$Z = \lambda^1_-(\mathbf{e}^1 + \mathbf{e}^{\bar{1}}) + \sqrt{2}\lambda^{\bar{1}}_-\mathbf{e}^2 \,, \tag{5.55}$$

and a vector orthogonal to V, W, Y and Z, as

$$X = i\lambda^{\bar{1}}_-(\mathbf{e}^1 - \mathbf{e}^{\bar{1}}) \,, \tag{5.56}$$

where X can also be shown to be Killing. Furthermore we find

$$dV = \frac{1}{f}(c_1 Y + c_2 Z) \wedge V \,, \tag{5.57}$$

$$dW = \frac{1}{f}(-c_1 Y + c_2 Z) \wedge W \,, \tag{5.58}$$

$$dX = -\frac{\omega_{1,\bar{1}2}\sqrt{2}}{\lambda^{\bar{1}}_{-}} Z \wedge X \,, \tag{5.59}$$

$$dY = -\frac{2\sqrt{2}c_1}{f} V \wedge W + \frac{c_2}{f} Z \wedge Y \,, \tag{5.60}$$

$$dZ = 0 \,, \tag{5.61}$$

$$d\lambda^{1}_{-} = -2\omega_{-,+1} Z \,, \tag{5.62}$$

$$d\lambda^{\bar{1}}_{-} = \frac{-1}{\sqrt{2}} \omega_{1,\bar{1}2} Z \ . \tag{5.63}$$

Here c_2 and f are given by

$$c_2 = \sqrt{2}\omega_{-,+1}\lambda^{1}_{-} + \omega_{-,+2}\lambda^{\bar{1}}_{-} \,, \tag{5.64}$$

$$f = \left(\frac{(\lambda^{1}_{-})^2 + (\lambda^{\bar{1}}_{-})^2}{\sqrt{2}} \right) , \tag{5.65}$$

$$df = c_2 Z \,, \tag{5.66}$$

and c_1, c_2, and f are related by

$$-c_1\lambda^{\bar{1}}_{-} + c_2\lambda^{1}_{-} = 2\omega_{-,+1} f \,, \tag{5.67}$$

$$c_1\lambda^{1}_{-} + c_2\lambda^{\bar{1}}_{-} = \sqrt{2}\omega_{-,+2} f \,, \tag{5.68}$$

$$(\omega_{-,+1} - \omega_{+,-1}) f = -c_1\lambda^{\bar{1}}_{-} \,, \tag{5.69}$$

$$(\omega_{-,+2} - \omega_{+,-2}) f = \sqrt{2} c_1 \lambda^{1}_{-} \ . \tag{5.70}$$

From (5.33), (5.34), and (5.40) we find that

$$c_2 = \chi V_I X^I \lambda^{\bar{1}}_{-} \,, \tag{5.71}$$

which together with (5.66) implies that

$$\partial_z f = \chi V_I X^I \lambda^{\bar{1}}_{-} \ . \tag{5.72}$$

In addition, (5.62) and (5.67) can be combined in the following way

$$d(f\lambda^{1}_{-}) = c_1 \lambda^{\bar{1}}_{-} Z \ . \tag{5.73}$$

The forms V, W, X, Y, and Z, can be expressed in terms of coordinates as

$$V = f_1 dv \,, \tag{5.74}$$

$$W = f_2 dw \,, \tag{5.75}$$

$$X = f_3 dx \,, \tag{5.76}$$

$$Y = f(dy + \beta) \,, \tag{5.77}$$

$$Z = dz \ . \tag{5.78}$$

The coordinate derivatives of the scalars (B.51) and (B.52) are

$$\partial_y X_I = 0 \,, \tag{5.79}$$

$$-\partial_z X_I = \frac{\chi}{f}(X_I V_J X^J - V_I)\lambda^{\bar{1}}_{-} \,, \tag{5.80}$$

which implies that

$$dX_I = -\frac{\chi}{f}(X_I V_J X^J - V_I)\lambda^{\bar{1}}_- Z \ . \tag{5.81}$$

The functions f_1, f_2, f_3 and the form β can be constrained, upon comparison with (5.57)–(5.61), by

$$d\log f_1 = c_1(dy+\beta) + d\log f + G dv \,, \tag{5.82}$$

$$d\log f_2 = -c_1(dy+\beta) + d\log f + H dw \,, \tag{5.83}$$

$$d\log f_3 = d\log(\lambda^{\bar{1}}_-)^2 \,, \tag{5.84}$$

$$d\beta = \frac{-2\sqrt{2}c_1 f_1 f_2}{f^2} dv\wedge dw \ . \tag{5.85}$$

We can rewrite these as

$$d\log\frac{f_1 f_2}{f^2} = G dv + H dw \,, \tag{5.86}$$

$$d\log\frac{f_1}{f_2} = 2c_1(dy+\beta) + G dv - H dw \,, \tag{5.87}$$

$$f_3 = c_3(\lambda^{\bar{1}}_-)^2 \ . \tag{5.88}$$

for c_3 a non-zero constant. Taking the exterior derivative of (5.87) and (5.86) we find respectively

$$2c_1 d\beta = (\partial_v H + \partial_w G) dv\wedge dw \,, \tag{5.89}$$

$$(-\partial_w G + \partial_v H) dv\wedge dw = 0 \ . \tag{5.90}$$

Upon comparing (5.89) with (5.85) we see that G and H have only a v and w dependance

$$\partial_v H = \partial_w G = \frac{-2\sqrt{2}(c_1)^2 f_1 f_2}{f^2} \,, \tag{5.91}$$

and satisfy

$$\partial_w\partial_v H = H\partial_v H \,, \tag{5.92}$$

$$\partial_v\partial_w G = G\partial_w G \ . \tag{5.93}$$

The field strength F^I takes the form

$$F^I = F^I_{12}\mathbf{e}^1\wedge\mathbf{e}^2 + F^I_{\bar{1}2}\mathbf{e}^{\bar{1}}\wedge\mathbf{e}^2 + F^I_{1\bar{1}}\mathbf{e}^1\wedge\mathbf{e}^{\bar{1}}, \tag{5.94}$$

with non-zero components, $F^I_{12}, F^I_{\bar{1}2}, F^I_{1\bar{1}}$, given by (4.2) and (4.6). These can be expressed in terms of the scalars using the scalar derivatives (5.59), together with (5.62) and (5.32), as

$$F^I = d\left(\frac{X^I\lambda^1_- c_3 dx}{\sqrt{2}}\right) \ . \tag{5.95}$$

The scalar derivatives (5.81) can in turn be put into the form

$$d(fX_I) = \chi V_I \lambda^{\bar{1}}_- Z \ . \tag{5.96}$$

using (5.72). To proceed we need to consider two cases depending on whether c_1 vanishes or not.

JHEP10(2008)103

5.1.1 Solutions with $c_1 = 0$

In the case that $c_1 = 0$, (5.73) reduces to

$$d(f\lambda_-^1) = 0\,, \tag{5.97}$$

so that

$$f\lambda_-^1 = c_4\,, \tag{5.98}$$

for non-zero constant c_4. Here f is implicitly related to the scalars via the relation

$$\partial_z(fX_I) = \chi V_I\left(\sqrt{2}f - \frac{c_4^2}{f^2}\right)^{\frac{1}{2}}\,. \tag{5.99}$$

We further find in this case that

$$d\log\frac{f_1}{f} = G dv\,, \tag{5.100}$$

$$d\log\frac{f_2}{f} = H dw\,, \tag{5.101}$$

and that the metric is given by

$$ds^2 = 2f(z)\left(\frac{f_1 f_2}{f^2}\right)dvdw - \frac{(c_3\lambda_-^{\bar{1}})^2}{2}dx^2 - \frac{1}{2\sqrt{2}f}dz^2 - \frac{f}{2\sqrt{2}}dy^2\,. \tag{5.102}$$

where

$$\lambda_-^{\bar{1}} = \left(\sqrt{2}f - \left(\frac{c_4}{f}\right)^2\right)^{\frac{1}{2}}\,. \tag{5.103}$$

Moreover, as G and $\frac{f_1}{f}$ can be seen to be functions of v, and H and $\frac{f_2}{f}$ are functions of w, we find that, for $c_1 = 0$, the 2-manifold given by, ${}^{(2)}ds^2 = 2(\frac{f_1 f_2}{f^2})dvdw$, is flat.

5.1.2 Solutions with $c_1 \neq 0$

On the other hand, if $c_1 \neq 0$ then (5.96), together with (5.73), can be explicitly integrated up to

$$X_I = \frac{1}{c_1}\left(\frac{K_I}{f} + \chi V_I\lambda_-^1\right)\,, \tag{5.104}$$

with K_I constant. The metric in our coordinates is now given, more generally, by

$$ds^2 = 2f(z)\left(\frac{f_1 f_2}{f^2}\right)dvdw - \frac{(c_3\lambda_-^{\bar{1}})^2}{2}dx^2 - \frac{1}{2\sqrt{2}f}dz^2 - \frac{f}{2\sqrt{2}}(dy+\beta)^2\,. \tag{5.105}$$

In this case we can relate the function $\frac{f_1 f_2}{f^2}$ to the Ricci scalar for the 2-manifold with metric, ${}^{(2)}ds^2 = 2\frac{f_1 f_2}{f^2}dvdw$. The Ricci scalar is given by

$$\begin{aligned} {}^{(2)}R &= \frac{-2}{(\frac{-\partial_v H}{2\sqrt{2}{c_1}^2})^3}\left(\frac{-1}{2\sqrt{2}{c_1}^2}\right)^2\left(\partial_v H\partial_v\partial_w\partial_v H - \partial_v\partial_v H\partial_w\partial_v H\right) \\ &= 4\sqrt{2}(c^1)^2\,, \end{aligned} \tag{5.106}$$

where we have made use of (5.92). This manifold is then found to be AdS_2. We can also make a gauge transformation $\beta \to \beta + d\log\frac{\tilde{f}_1}{\tilde{f}_2}$, to eliminate the x and z dependance of β. The y dependance of f_1, f_2 can further be expressed as

$$f_1 = \tilde{f}_1 \exp(c_1 y)\,, \tag{5.107}$$
$$f_2 = \tilde{f}_2 \exp(-c_1 y)\,, \tag{5.108}$$

so that (5.87) reduces to

$$\beta = H dw - G dv\,, \tag{5.109}$$

where β is only a function of v and w.

5.2 Solutions with $\lambda^1_- = 0$ and $\lambda^{\bar{1}}_- \neq 0$

The $\lambda^{\bar{1}}_-$ derivatives are

$$\partial_+ \lambda^{\bar{1}}_- = -\omega_{+,1\bar{1}} \lambda^{\bar{1}}_-\,, \tag{5.110}$$
$$\partial_- \lambda^{\bar{1}}_- = (-\omega_{-,1\bar{1}} + \omega_{\bar{1},-1})\lambda^{\bar{1}}_-, \tag{5.111}$$
$$\partial_1 \lambda^{\bar{1}}_- = -\omega_{1,1\bar{1}} \lambda^{\bar{1}}_-\,, \tag{5.112}$$
$$\partial_{\bar{1}} \lambda^{\bar{1}}_- = -\omega_{\bar{1},1\bar{1}} \lambda^{\bar{1}}_-\,, \tag{5.113}$$
$$\partial_2 \lambda^{\bar{1}}_- = (\omega_{-,+2} - \omega_{2,1\bar{1}})\lambda^{\bar{1}}_-\,, \tag{5.114}$$

and the other non-zero components of the spin connection are related by

$$\omega_{-,+2} = \omega_{+,-2} = -\omega_{1,\bar{1}2} = \chi V_I X^I\,, \tag{5.115}$$
$$\omega_{\bar{1},-1} = -\frac{1}{2}\omega_{2,-2}\,. \tag{5.116}$$

We find for the scalars

$$dX^I = 2\chi\left(X^I V_J X^J - \frac{3}{2} Q^{IJ} V_J\right) e^2\,, \tag{5.117}$$

and gauge potential

$$\chi A_- = \frac{i}{3}\omega_{-,1\bar{1}} - i\omega_{\bar{1},-1}\,. \tag{5.118}$$

We can use a gauge transformation as in (5.20), taking

$$\psi^{\bar{1}}_\pm \to e^{2ig'\mu}\psi^{\bar{1}}_\pm\,, \tag{5.119}$$

to set $\lambda^{\bar{1}}_- \in \mathbb{R}$. As a result we find

$$\omega_{+,1\bar{1}} = \omega_{-,1\bar{1}} = \omega_{1,1\bar{1}} = \omega_{2,1\bar{1}} = 0\,, \tag{5.120}$$

and

$$d\lambda^{\bar{1}}_- = \omega_{-,+2}\lambda^{\bar{1}}_- \mathbf{e}^2\,. \tag{5.121}$$

The field strength F^I vanishes in this case. We find closed forms

$$V = \mathbf{e}^+\,, \tag{5.122}$$
$$W = h^{-1}\mathbf{e}^-\,, \tag{5.123}$$

$$X = (\sqrt{2}h)^{-\frac{1}{2}}(\mathbf{e}^1 + \mathbf{e}^{\bar{1}})\,, \tag{5.124}$$
$$Y = i(\sqrt{2}h)^{-\frac{1}{2}}(\mathbf{e}^1 - \mathbf{e}^{\bar{1}})\,, \tag{5.125}$$
$$Z = \mathbf{e}^2, \tag{5.126}$$

where $h = (\lambda^{\bar{1}}_-)^2$. Then specify a coordinate basis

$$V = dv, W = dw, X = dx, Y = dy, Z = dz\ . \tag{5.127}$$

In this basis

$$dh = 2\chi V_I X^I h dz\,, \tag{5.128}$$

so that upon comparison with (5.117), we find that

$$\partial_z(hX_I) = 2\chi V_I h\ . \tag{5.129}$$

The metric is given by

$$ds^2 = h(2dvdw - dx^2 - dy^2) - dz^2\ . \tag{5.130}$$

5.3 Solutions with $\lambda^1_- \neq 0$ and $\lambda^{\bar{1}}_- = 0$

The λ^1_- derivatives vanish in this case

$$d\lambda^1_- = 0\ . \tag{5.131}$$

The following components of the spin connection vanish

$$\omega_{-,+1} = \omega_{+,-1} = \omega_{1,\bar{1}2} = 0\,, \tag{5.132}$$
$$\omega_{1,-1} = \omega_{2,-1} = \omega_{-,-2} = \omega_{-,-1} = \omega_{-,12} = \omega_{1,-2} = 0\,, \tag{5.133}$$

and we have

$$\omega_{-,+2} = -\omega_{+,-2}\,, \tag{5.134}$$
$$\omega_{\bar{1},-1} = -\frac{1}{2}\omega_{2,-2} = 0\ . \tag{5.135}$$

We also find that the scalars are constant

$$dX^I = 0\,, \tag{5.136}$$

and for the gauge potential

$$\chi A = \frac{i}{3}\omega_{+,1\bar{1}}\mathbf{e}^+ + \frac{i}{3}\omega_{-,1\bar{1}}\mathbf{e}^- + \frac{i}{3}\omega_{1,1\bar{1}}\mathbf{e}^1 + \frac{i}{3}\omega_{\bar{1},1\bar{1}}\mathbf{e}^{\bar{1}} + \frac{i}{3}\omega_{2,1\bar{1}}\mathbf{e}^2\ . \tag{5.137}$$

We can integrate up the scalars, in the process defining a constant c by

$$\chi V_I X^I = c = \frac{2}{3}\omega_{-,+2}\,, \tag{5.138}$$

with $X^I = q^I$. The field strengths have non-vanishing component

$$F^I_{1\bar{1}} = 3i\chi(-X^I V_J X^J + Q^{IJ} V_J)\,, \tag{5.139}$$

which are therefore also constants. We can contract this with χV_I, to find

$$F = \chi V_I F^I = ik\mathbf{e}^1 \wedge \mathbf{e}^{\bar{1}}\,, \tag{5.140}$$

with constant $k = -3\big(c^2 - \chi^2 Q^{IJ} V_I V_J)\big)$.

Taking the exterior derivative of the basis forms, one obtains

$$d\mathbf{e}^+ = -3c\mathbf{e}^2 \wedge \mathbf{e}^+\,, \tag{5.141}$$
$$d\mathbf{e}^- = 3c\mathbf{e}^2 \wedge \mathbf{e}^-\,, \tag{5.142}$$
$$d\mathbf{e}^1 = 3i\chi A \wedge \mathbf{e}^1\,, \tag{5.143}$$
$$d\mathbf{e}^{\bar{1}} = -3i\chi A \wedge \mathbf{e}^{\bar{1}}\,, \tag{5.144}$$
$$d\mathbf{e}^2 = 3c\mathbf{e}^+ \wedge \mathbf{e}^-\,. \tag{5.145}$$

Coordinates can be introduced for $\mathbf{e}^+$ and $\mathbf{e}^-$ as

$$\mathbf{e}^+ = g_1 dv\,, \tag{5.146}$$
$$\mathbf{e}^- = g_2 dw\,. \tag{5.147}$$

Comparing (5.141) and (5.142) with (5.146), (5.147), we find

$$d\log g_1 = -3c\mathbf{e}^2 + 3c\alpha_1 dv\,, \tag{5.148}$$
$$d\log g_2 = 3c\mathbf{e}^2 - 3c\alpha_2 dw\,, \tag{5.149}$$

for some real functions α_1, α_2. These can be rewritten as

$$d\log g_1 g_2 = 3c(\alpha_1 dv - \alpha_2 dw)\,, \tag{5.150}$$
$$d\log \frac{g_1}{g_2} = -6c\mathbf{e}^2 + 3c(\alpha_1 dv + \alpha_2 dw)\,. \tag{5.151}$$

Then (5.151) defines $\mathbf{e}^2$ implicitly to be

$$\mathbf{e}^2 = dz + \frac{1}{2}(\alpha_1 dv + \alpha_2 dw)\,, \tag{5.152}$$

where we define the coordinate z, such that, $dz = \frac{-1}{6c} d\log\frac{g_1}{g_2}$. Next we can introduce complex coordinates for $\mathbf{e}^1, \mathbf{e}^{\bar{1}}$ as

$$\mathbf{e}^1 = s d\ell\,, \tag{5.153}$$
$$\mathbf{e}^{\bar{1}} = \bar{s} d\bar{\ell}\,, \tag{5.154}$$

where $s = re^{i\theta}$, and $d\ell = dx + idy$. Then

$$d\log s + q d\ell = 3i\chi A\,, \tag{5.155}$$
$$d\log \bar{s} + \bar{q} d\bar{\ell} = -3i\chi A\,, \tag{5.156}$$

upon comparison with (5.143) and (5.144). Here q is a complex function $q = q_1 + iq_2$. These expressions can in turn be rewritten as

$$d\log s\bar{s} = -qd\ell - \bar{q}d\bar{\ell}\,, \tag{5.157}$$

$$d\log\frac{s}{\bar{s}} = 6i\chi A + \bar{q}d\bar{\ell} - qd\ell\ . \tag{5.158}$$

(5.158) implicitly defines A, up to a gauge transformation, as

$$\chi A = \frac{1}{3}(q_2 dx + q_1 dy)\ . \tag{5.159}$$

With these coordinates, the metric takes the form

$$ds^2 = 2g_1 g_2 dvdw - \left(dz + \frac{\alpha}{2}\right)^2 - 2r^2(dx^2 + dy^2)\,, \tag{5.160}$$

where $\alpha = \alpha_1 dv + \alpha_2 dw$. We can proceed to investigate the curvature of the 3-manifold with metric

$${}^{(3)}ds^2 = 2g_1 g_2 dvdw - \left(dz + \frac{\alpha}{2}\right)^2\ . \tag{5.161}$$

To do this we take the exterior derivative of (5.150) and (5.151)

$$d\alpha_1 \wedge dv - d\alpha_2 \wedge dw = 0\,, \tag{5.162}$$

$$d\mathbf{e}^2 = \frac{1}{2}(d\alpha_1 \wedge dv + d\alpha_2 \wedge dw)\ . \tag{5.163}$$

These constraints, together with (5.145) imply

$$\partial_v \alpha_2 = -\partial_w \alpha_1 = 3cg_1 g_2\,, \tag{5.164}$$

and that $\alpha_1 = \alpha_1(v,w), \alpha_2 = \alpha_2(v,w)$. Substituting this back into the expression (5.150) for $g_1 g_2$, we see that

$$\frac{d(\partial_v \alpha_2)}{\partial_v \alpha_2} = 3c(\alpha_1 dv - \alpha_2 dw)\ . \tag{5.165}$$

Next we note that the 2-manifold with metric, ${}^{(2)}ds^2 = 2g_1 g_2 dvdw$ is AdS_2 with Ricci scalar $18c^2$, and that α is related to the volume form for this manifold by $d\alpha = 6c\ \mathrm{dvol}(AdS_2)$. It then follows that the 3-manifold with metric (5.161) is AdS_3 (written as a fibration over AdS_2), with Ricci scalar

$${}^{(3)}R = \frac{27c^2}{2}\ . \tag{5.166}$$

We can, in a similar manner, compute the Ricci scalar for the 2-manifold with metric ${}^{(2)}ds^2 = 2s\bar{s}d\ell d\bar{\ell} = 2r^2 d\ell d\bar{\ell}$.

Taking the exterior derivative of (5.157) and (5.158) provides

$$dq \wedge d\ell + d\bar{q} \wedge d\bar{\ell} = 0\,, \tag{5.167}$$

$$6i\chi dA = dq \wedge d\ell - d\bar{q} \wedge d\bar{\ell}\ . \tag{5.168}$$

Given that $F = \chi dA$ we can compare this with (5.140), to find

$$\partial_{\bar{\ell}} q = 3kr^2 \,, \tag{5.169}$$

where $r^2 = s\bar{s}$. If we substitute this back into the expression (5.157) for $s\bar{s}$, we find that

$$\frac{d(\partial_{\bar{\ell}} q)}{\partial_{\bar{\ell}} q} = -q d\ell - \bar{q} d\bar{\ell} \,. \tag{5.170}$$

The Ricci scalar is given by (making use of (5.170))

$$\begin{aligned} {}^{(2)}R &= \frac{-2}{(\frac{1}{3k}\partial_{\bar{\ell}} q)^3}\left(\frac{1}{3k}\right)^2 \left(\partial_{\bar{\ell}} q \partial_\ell \partial_{\bar{\ell}} \partial_{\bar{\ell}} q - \partial_{\bar{\ell}} \partial_{\bar{\ell}} q \partial_\ell \partial_{\bar{\ell}} q\right) \\ &= 6k \,. \end{aligned} \tag{5.171}$$

The 2-manifold is then $\mathbb{H}^2$, $\mathbb{R}^2$, or S^2 according as to whether the constant $k = -3(c^2 - \chi^2 Q^{IJ} V_I V_J)$ is negative, vanishing, or positive respectively.

6. Solutions with $\boldsymbol{\lambda^\alpha_- = 0}$

For these solutions, we have Killing spinors $\eta_1 = \psi^1_+$ and $\eta_2 = \lambda^1_+ \psi^1_+ + \lambda^{\bar{1}}_+ \psi^{\bar{1}}_+$. Note that if $\lambda^{\bar{1}}_+ = 0$, then requiring that both η_1, η_2 satisfy the Killing spinor equations forces λ^1_+ to be constant; such solutions are not half-supersymmetric. Hence, we must take $\lambda^{\bar{1}}_+ \neq 0$. It is straightforward to show that the stability subgroup is

$$\mathrm{Stab}(\eta_1, \eta_2) = \mathbb{R}^3 \tag{6.1}$$

where $\mathbb{R}^3$ is generated by $\{\Gamma_{+1}, \Gamma_{+\bar{1}}, \Gamma_{+2}\}$. The Σ-group is then

$$\Sigma(\mathcal{P}) = \mathrm{Spin}(1,1) \times su(2) \tag{6.2}$$

where $\mathrm{Spin}(1,1)$ is generated by Γ_{+-} and $su(2)$ is generated by $\{\Gamma_{12}, \Gamma_{\bar{1}2}, \Gamma_{1\bar{1}}\}$.

To proceed, we analyse the dilatino and gravitino equations; for the dilatino equations:

$$8i\chi\left(X^I V_J X^J - \frac{3}{2} Q^{IJ} V_J\right)\lambda^{\bar{1}}_+ = 0 \,. \tag{6.3}$$

For the gravitino equations, in the + direction

$$\partial_+ \lambda^1_+ = 0 \,, \tag{6.4}$$

$$\partial_+ \lambda^{\bar{1}}_+ + \omega_{+,1\bar{1}} \lambda^{\bar{1}}_+ = 0 \,. \tag{6.5}$$

In the − direction

$$\partial_- \lambda^1_+ = 0 \,, \tag{6.6}$$

$$(\partial_- - 3i\chi A_-)\lambda^{\bar{1}}_+ + \omega_{-,1\bar{1}} \lambda^{\bar{1}}_+ = 0 \,, \tag{6.7}$$

$$\sqrt{2} i\chi V_I X^I \lambda^{\bar{1}}_+ = 0 \,. \tag{6.8}$$

In the 1 direction

$$\partial_1 \lambda^1_+ = 0\,, \tag{6.9}$$

$$\partial_1 \lambda^{\bar{1}}_+ + \omega_{1,1\bar{1}} \lambda^{\bar{1}}_+ - 2\omega_{1,+-} \lambda^{\bar{1}}_+ = 0\ . \tag{6.10}$$

In the $\bar{1}$ direction

$$\partial_{\bar{1}} \lambda^1_+ + \chi\sqrt{2} V_I X^I \lambda^{\bar{1}}_+ = 0\,, \tag{6.11}$$

$$\partial_{\bar{1}} \lambda^{\bar{1}}_+ + 2\omega_{\bar{1},+-} \lambda^{\bar{1}}_+ + \omega_{\bar{1},1\bar{1}} \lambda^{\bar{1}}_+ = 0\ . \tag{6.12}$$

In the 2 direction

$$\partial_2 \lambda^1_+ = 0\,, \tag{6.13}$$

$$\partial_2 \lambda^{\bar{1}}_+ + \chi V_I X^I \lambda^{\bar{1}}_+ + \omega_{2,1\bar{1}} \lambda^{\bar{1}}_+ = 0\ . \tag{6.14}$$

These constraints imply that $\lambda^{\bar{1}}_+ = 0$ and that λ^1_+ is constant. Hence, as mentioned previously, these solutions are in fact only 1/4 supersymmetric.

7. Summary of results

In this paper we examined half supersymmetric solutions of gauged $N = 2, D = 5$ supergravity coupled to an arbitrary number of abelian vector multiplets for which the Killing vectors obtained as bilinears from the Killing spinors are all null. This analysis completes the work initiated in [2], where half-supersymmetric solutions with at least one timelike Killing vector were systematically classified. We have also shown that the integrability constraints imposed by the Killing spinor equations, together with the Bianchi identity for the 2-form field strengths, are sufficient to imply that the Einstein, gauge and scalar equations hold automatically.

Four classes of solutions were obtained from this analysis:

(i) In the case where $(\lambda^1_- \neq 0, \lambda^{\bar{1}}_- \neq 0, c_1 \neq 0)$ the metric is given by

$$ds^2 = f ds^2(AdS_2) - (\lambda^{\bar{1}}_-)^2 dx^2 - \frac{1}{2\sqrt{2} f} dz^2 - \frac{f}{2\sqrt{2}} (dy + \beta)^2\,, \tag{7.1}$$

where $ds^2(AdS_2)$ has Ricci scalar $R_{AdS_2} = 4\sqrt{2} c_1^2$. β is a one form on AdS_2 with

$$d\beta = -2\sqrt{2} c_1 \ \mathrm{dvol}(AdS_2)\ . \tag{7.2}$$

Here c_1 is a non-zero constant, and $\lambda^1_-, \lambda^{\bar{1}}_- \in \mathbb{R}$. We also find that f, $\lambda^1_-, \lambda^{\bar{1}}_-$ and the scalars X^I are functions of z constrained by

$$X_I = \frac{1}{c_1} \left(\frac{K_I}{f} + \chi V_I \lambda^1_- \right), \tag{7.3}$$

$$f = \frac{((\lambda^{\bar{1}}_-)^2 + (\lambda^1_-)^2)}{\sqrt{2}}\,, \tag{7.4}$$

$$\partial_z (f \lambda^1_-) = c_1 \lambda^{\bar{1}}_-\,, \tag{7.5}$$

for constant K_I. It does not appear to be possible to de-couple these equations in general. The field strengths F^I satisfy

$$F^I = d(X^I \lambda^1_- dx) \ . \tag{7.6}$$

We remark that although it would appear that these solutions depend on a free parameter c_1, we can without loss of generality set $c_1 = 1$. This can be achieved by making the re-scalings

$$\begin{aligned} &\lambda^1_- = c_1(\lambda^1_-)', \qquad &&\lambda^{\bar{1}}_- = c_1(\lambda^{\bar{1}}_-)', \qquad &&f = c_1^2 f', \qquad &&K_I = c_1^3 (K_I)' \\ &z = c_1 z', \qquad &&x = \frac{1}{c_1} x', \qquad &&y = \frac{1}{c_1} y', \qquad &&\beta = \frac{1}{c_1}\beta' \end{aligned} \tag{7.7}$$

and defining the conformally re-scaled AdS_2 factor by

$$ds^2(AdS_2') = c_1^2 ds^2(AdS_2) \tag{7.8}$$

so that $R_{AdS_2'} = 4\sqrt{2}$, and $d\beta' = -2\sqrt{2}$ dvol(AdS_2'). On dropping the primes, it is clear that one can set $c_1 = 1$ without loss of generality.

(ii) In the case that $(\lambda^1_- \neq 0, \lambda^{\bar{1}}_- \neq 0, c_1 = 0)$ we find for the metric

$$ds^2 = f ds^2(\mathbb{R}^{1,1}) - \left(\sqrt{2} f - \frac{c_4^2}{f^2}\right) dx^2 - \frac{1}{2\sqrt{2} f} dz^2 - \frac{f}{2\sqrt{2}} dy^2 \,, \tag{7.9}$$

for non-zero constant c_4. Here the function f and the scalars X^I are constrained by

$$\partial_z (f X_I) = \chi V_I \left(\sqrt{2} f - \frac{c_4^2}{f^2}\right)^{\frac{1}{2}} \,, \tag{7.10}$$

and the field strengths F^I are given by

$$F^I = c_4 \ d\left(\frac{X^I}{f} dx\right) \ . \tag{7.11}$$

(iii) In the case that $(\lambda^1_- = 0, \lambda^{\bar{1}}_- \neq 0)$, we find that the field strengths vanish, $F^I = 0$. In addition, the metric is given by

$$ds^2 = h \ ds^2(\mathbb{R}^{1,3}) - dz^2 \,, \tag{7.12}$$

and the scalars satisfy

$$\partial_z (h X_I) = 2\chi V_I h \ . \tag{7.13}$$

where $h = (\lambda^{\bar{1}}_-)^2$. This can be seen to be the domain wall solution found in [10], where we identify $h = (\partial_u f)^{\frac{2}{3}}$, and $\chi = g$. Note that these solutions can be obtained from the type (ii) solution described above, by taking the limit $c_4 \to 0$.

(iv) In the case that ($\lambda^1_- \neq 0, \lambda^{\bar{1}}_- = 0$) we find that the scalars X^I are constant, and the metric is

$$ds^2 = ds^2(AdS_3) - ds^2(M_2) \ . \tag{7.14}$$

where M_2 is a 2-manifold with Ricci scalar

$$R_{M_2} = -18\chi^2(X^I X^J - Q^{IJ})V_I V_J \tag{7.15}$$

so M_2 is $\mathbb{H}^2$, $\mathbb{R}^2$, or S^2 according as to whether $(X^I X^J - Q^{IJ})V_I V_J$ is positive, zero or negative. Note that in the minimal theory $(X^I X^J - Q^{IJ})V_I V_J = (V_1)^2$, so the cases for which M_2 is $\mathbb{R}^2$ or S^2 cannot arise in the minimal theory.

The AdS_3 manifold has Ricci scalar

$$R_{AdS_3} = \frac{27\chi^2 V_I V_J X^I X^J}{2} \ . \tag{7.16}$$

For the field strengths we find

$$F^I = -3\chi(-X^I V_J X^J + Q^{IJ}V_J)\ \mathrm{dvol}(M_2) \ . \tag{7.17}$$

Note that these product space solutions have previously been found in the context of black string solutions constructed in [6] and [7].

7.1 Interpretation of solutions

As we have already stated, the solutions (iii) correspond to domain wall solutions found in [10], and the solutions (iv) correspond to near horizon black string solutions [6] and [7]. We shall therefore concentrate on solutions (i) and (ii). We shall further assume that the scalar manifold is symmetric, in which case one has the identity

$$\frac{9}{2}C^{IJK}X_I X_J X_K = 1 \tag{7.18}$$

where

$$C^{IJK} = \delta^{II'}\delta^{JJ'}\delta^{KK'}C_{I'J'K'} \ . \tag{7.19}$$

It is then possible to construct the metrics explicitly. We begin with the solutions of type (i). As mentioned previously, we shall set $c_1 = 1$ without loss of generality.To proceed, it is convenient to set

$$\xi^3 = \frac{9}{2}C^{IJK}V_I V_J V_K \tag{7.20}$$

and we assume that $\xi \neq 0$. Also define $\hat{\rho}_I$, $\hat{x}$ by

$$\begin{aligned} K_I &= 2\sqrt{2}C^{-2}\hat{\rho}_I \\ f\lambda^1_- &= \frac{2\sqrt{2}C^{-2}}{\chi\xi}\hat{x} \end{aligned} \tag{7.21}$$

for constant $C > 0$. Note that $\hat{x}$ is not constant. Also set

$$\begin{aligned}\hat{\alpha}_0 &= \frac{9}{2}C^{IJK}\hat{\rho}_I\hat{\rho}_J\hat{\rho}_K \\ \hat{\alpha}_1 &= \frac{9}{2\xi}C^{IJK}\hat{\rho}_I\hat{\rho}_J V_K \\ \hat{\alpha}_2 &= \frac{9}{2\xi^2}C^{IJK}\hat{\rho}_I V_J V_K \ .\end{aligned} \tag{7.22}$$

Then (7.3) implies that

$$f = 2\sqrt{2}C^{-2}H^{\frac{1}{3}} \tag{7.23}$$

where

$$H = \hat{x}^3 + 3\hat{\alpha}_2\hat{x}^2 + 3\hat{\alpha}_1\hat{x} + \hat{\alpha}_0 \tag{7.24}$$

so that the scalars satisfy

$$H^{\frac{1}{3}}X_I = \hat{\rho}_I + \frac{V_I}{\xi}\hat{x} \ . \tag{7.25}$$

It is also convenient to introduce co-ordinates v, ρ and write the AdS_2 factor in the metric as

$$ds^2(AdS_2) = -\frac{C^2}{\sqrt{2}}dvd\rho + \frac{C^4}{2\sqrt{2}}\rho^2 dv^2 \ . \tag{7.26}$$

Finally, on making the re-scalings

$$x = \chi\xi\hat{x}^2, \quad y = C^2\hat{x}^1, \quad \beta = C^2\hat{\beta} \tag{7.27}$$

and using (7.5), one can rewrite the metric as

$$\begin{aligned}ds^2 \ = \ & H^{\frac{1}{3}}(-2dvd\rho + C^2c_1^2\rho^2dv^2) - \frac{4(\chi\xi)^2}{C^2}H^{-\frac{2}{3}}P(d\hat{x}^2)^2 \\ & -\frac{H^{\frac{1}{3}}}{4(\chi\xi)^2}P^{-1}(d\hat{x})^2 - C^2H^{\frac{1}{3}}(d\hat{x}^1 + \hat{\beta})^2\end{aligned} \tag{7.28}$$

where

$$\hat{\beta} = \rho dv, \quad P = H - \frac{C^2}{4(\chi\xi)^2}\hat{x}^2 \ . \tag{7.29}$$

Finally, define a radial co-ordinate r by

$$r = H^{\frac{1}{3}}\rho \ . \tag{7.30}$$

It is then straightforward to see that this metric corresponds to one of the three classes of "static" local near horizon geometries, written in Gaussian null co-ordinates, as constructed in [25] (on dropping the $\hat{}$ on x, x^1 and x^2). The horizon is at $r = 0$. Note that one can set $C = 1$ without loss of generality, by making appropriately chosen re-scalings, however we

retain C here for ease of comparison. Furthermore, one can also set $\hat{\alpha}_2 = 0$ by making a constant shift in the $\hat{x}$ co-ordinate, this then produces a modification to the function P. It should be noted that a global analysis was carried out in [25] which showed that the spatial cross-sections of the horizon cannot be regular and compact.

The analysis of the type (ii) solutions is somewhat more straightforward. In particular, define

$$\xi^3 = \frac{9}{2} C^{IJK} V_I V_J V_K \tag{7.31}$$

and again assume that $\xi \neq 0$, also set

$$\begin{aligned} \alpha_0 &= \frac{9}{2} C^{IJK} \rho_I \rho_J \rho_K \\ \alpha_1 &= \frac{9}{2\xi} C^{IJK} \rho_I \rho_J V_K \\ \alpha_2 &= \frac{9}{2\xi^2} C^{IJK} \rho_I V_J V_K \ . \end{aligned} \tag{7.32}$$

It is also convenient to define $\hat{x}$ such that

$$\frac{d\hat{x}}{dz} = \chi\xi \sqrt{\sqrt{2} f - \frac{c_4^2}{f^2}} \tag{7.33}$$

so

$$f X_I = \rho_I + \frac{V_I}{\xi} \hat{x} \tag{7.34}$$

and hence

$$f = \left(\hat{x}^3 + 3\alpha_2 \hat{x}^2 + 3\alpha_1 \hat{x} + \alpha_0 \right)^{\frac{1}{3}} \ . \tag{7.35}$$

Also define $\hat{x}^1, \hat{x}^2, \tau_0$ by

$$\begin{aligned} x &= 2^{-\frac{1}{4}} \hat{x}^2 \\ y &= 2^{\frac{3}{4}} C \hat{x}^1 \\ c_4^2 &= \sqrt{2} \tau_0^3 \end{aligned} \tag{7.36}$$

for constant $C > 0$. Then the metric can be written as

$$ds^2 = 2f dv d\rho - f^{-2}(f^3 - \tau_0^3)(d\hat{x}^2)^2 - \frac{f}{4(\xi\chi)^2}(f^3 - \tau_0^3)^{-1}(d\hat{x})^2 - C^2 f (d\hat{x}^1)^2 \ . \tag{7.37}$$

On defining the radial co-ordinate r by

$$r = f^{\frac{1}{3}} \rho \tag{7.38}$$

we recover the second type of "static" near horizon geometry constructed in [25], in the case for which $\Gamma_0 > 0$. The static solutions with $\Gamma_0 = 0$ found in that paper correspond to the type (iii) domain wall solutions, with symmetric scalar manifold. Once more, a

global analysis has been constructed, which shows that these solutions do not correspond to compact near horizon geometries of regular black holes. We also remark that it is straightforward to prove that the solutions of type (iv) correspond to the "static" solutions with constant scalars found in [25].

To summarize, we have shown that when the scalar manifold is symmetric, and when $C^{IJK}V_IV_JV_K \neq 0$,[1] the set of static solutions found in [25] for which the Killing vector generated from the Killing spinor is null (excluding the trivial case of the maximally supersymmetric solution AdS_5) is identical to the set of all half supersymmetric solutions for which all of the Killing spinors generate null Killing vectors.

A. Integrability conditions

The gravitino and dilatino integrability conditions, respectively, can be put into the form

$$\left(E_\alpha{}^\beta\Gamma_\beta + \frac{1}{3}G^\beta\Gamma_{\alpha\beta} - \frac{2}{3}G_\alpha\right)\epsilon = 0\,, \tag{A.1}$$

$$\left(S_I - \frac{2}{3}(G_{I\alpha} - X_IX^JG_{J\alpha})\Gamma^\alpha\right)\epsilon = 0\,, \tag{A.2}$$

acting on a Dirac spinor $\epsilon = \lambda^1_+\psi^1_+ + \lambda^{\bar{1}}_+\psi^{\bar{1}}_+ + \lambda^1_-\psi^1_- + \lambda^{\bar{1}}_-\psi^{\bar{1}}_-$. Here

$$\begin{aligned} E_{\alpha\beta} &= R_{\alpha\beta} + Q_{IJ}F^I{}_{\alpha\mu}F^J{}_\beta{}^\mu - Q_{IJ}\nabla_\alpha X^I\nabla_\beta X^J \\ &\quad + g_{\alpha\beta}\bigg(-\frac{1}{6}Q_{IJ}F^I_{\beta_1\beta_2}F^{J\beta_1\beta_2} + 6\chi^2\bigg(\frac{1}{2}Q^{IJ} - X^IX^J\bigg)V_IV_J\bigg)\,, \end{aligned} \tag{A.3}$$

$$G_{I\alpha} = \nabla^\beta(Q_{IJ}F^J{}_{\alpha\beta}) + \frac{1}{16}C_{IJK}\epsilon_\alpha{}^{\beta_1\beta_2\beta_3\beta_4}F^J{}_{\beta_1\beta_2}F^K{}_{\beta_3\beta_4}\,, \tag{A.4}$$

$$\begin{aligned} S_I &= \nabla^\alpha\nabla_\alpha X_I - \left(\frac{1}{6}C_{MNI} - \frac{1}{2}X_IC_{MNJ}X^J\right)\nabla_\alpha X^M\nabla^\alpha X^N \\ &\quad -\frac{1}{2}\bigg(X_MX^PC_{NPI} - \frac{1}{6}C_{MNI} - 6X_IX_MX_N + \frac{1}{6}X_IC_{MNJ}X^J\bigg)F^M{}_{\beta_1\beta_2}F^{N\beta_1\beta_2} \\ &\quad -3\chi^2V_MV_N\bigg(\frac{1}{2}Q^{ML}Q^{NP}C_{LPI} + X_I(Q^{MN} - 2X^MX^N)\bigg)\,, \end{aligned} \tag{A.5}$$

with $G_\beta = X^IG_{I\beta}$. The $\psi^1_+, \psi^{\bar{1}}_+, \psi^1_-, \psi^{\bar{1}}_-$ components of (A.1) are respectively, for $\alpha = +$

$$\sqrt{2}E_{+-}\lambda^1_- + \sqrt{2}iE_{+1}\lambda^{\bar{1}}_+ - iE_{+2}\lambda^1_+ + \frac{1}{3}(-G_+\lambda^1_+ - 2iG_1\lambda^{\bar{1}}_- + \sqrt{2}iG_2\lambda^1_-) = 0\,, \tag{A.6}$$

$$\sqrt{2}E_{+-}\lambda^{\bar{1}}_- + \sqrt{2}iE_{+\bar{1}}\lambda^1_+ - iE_{+2}\lambda^{\bar{1}}_+ - \frac{1}{3}(G_+\lambda^{\bar{1}}_+ + 2iG_{\bar{1}}\lambda^1_- + \sqrt{2}iG_2\lambda^{\bar{1}}_-) = 0\,, \tag{A.7}$$

$$\sqrt{2}E_{++}\lambda^1_+ - \sqrt{2}iE_{+1}\lambda^{\bar{1}}_- + iE_{+2} - \lambda^1_- - G_+\lambda^1_- = 0\,, \tag{A.8}$$

$$\sqrt{2}E_{++}\lambda^{\bar{1}}_+ - \sqrt{2}iE_{+\bar{1}}\lambda^1_- - iE_{+2} - \lambda^{\bar{1}}_- - G_+\lambda^{\bar{1}}_- = 0\,. \tag{A.9}$$

For $\alpha = -$

$$\sqrt{2}E_{--}\lambda^1_- + \sqrt{2}iE_{-1}\lambda^{\bar{1}}_+ - iE_{-2}\lambda^1_+ - G_-\lambda^1_+ = 0\,, \tag{A.10}$$

[1] This condition holds for all solutions of the minimal theory, and also for all asymptotically AdS_5 solutions.

JHEP10(2008)103

$$\sqrt{2}E_{--}\lambda^{\bar{1}}_{-} + \sqrt{2}iE_{-\bar{1}}\lambda^{1}_{+} - iE_{-2}\lambda^{\bar{1}}_{+} - G_{-}\lambda^{\bar{1}}_{+} = 0\,, \quad \text{(A.11)}$$

$$\sqrt{2}E_{-+}\lambda^{1}_{+} - \sqrt{2}iE_{-1}\lambda^{\bar{1}}_{-} + iE_{-2}\lambda^{1}_{-} + \frac{1}{3}(-G_{-}\lambda^{1}_{-} + 2iG_{1}\lambda^{\bar{1}}_{+} - \sqrt{2}iG_{2}\lambda^{1}_{+}) = 0\,, \quad \text{(A.12)}$$

$$\sqrt{2}E_{-+}\lambda^{\bar{1}}_{+} - \sqrt{2}iE_{-\bar{1}}\lambda^{1}_{-} + iE_{-2}\lambda^{\bar{1}}_{-} + \frac{1}{3}(-G_{-}\lambda^{\bar{1}}_{-} + 2iG_{\bar{1}}\lambda^{1}_{+} + \sqrt{2}iG_{2}\lambda^{\bar{1}}_{+}) = 0\,. \quad \text{(A.13)}$$

For $\alpha = 1$

$$\sqrt{2}E_{1-}\lambda^{1}_{-} + \sqrt{2}iE_{11}\lambda^{\bar{1}}_{+} - iE_{12}\lambda^{1}_{+} - G_{1}\lambda^{1}_{+} = 0\,, \quad \text{(A.14)}$$

$$\sqrt{2}E_{1-}\lambda^{\bar{1}}_{-} + \sqrt{2}iE_{1\bar{1}}\lambda^{1}_{+} + iE_{12}\lambda^{\bar{1}}_{+} + \frac{1}{3}(-2iG_{-}\lambda^{1}_{-} - \sqrt{2}G_{2}\lambda^{1}_{+} - G_{1}\lambda^{\bar{1}}_{+}) = 0\,, \quad \text{(A.15)}$$

$$\sqrt{2}E_{1+}\lambda^{1}_{+} - \sqrt{2}iE_{11}\lambda^{\bar{1}}_{-} + iE_{12}\lambda^{1}_{-} - G_{1}\lambda^{1}_{-} = 0\,, \quad \text{(A.16)}$$

$$\sqrt{2}E_{1+}\lambda^{\bar{1}}_{+} - \sqrt{2}iE_{1\bar{1}}\lambda^{1}_{-} - iE_{12}\lambda^{\bar{1}}_{-} + \frac{1}{3}(2iG_{+}\lambda^{1}_{+} - \sqrt{2}G_{2}\lambda^{1}_{-} - G_{1}\lambda^{\bar{1}}_{-}) = 0\,. \quad \text{(A.17)}$$

For $\alpha = \bar{1}$

$$\sqrt{2}E_{\bar{1}-}\lambda^{1}_{-} + \sqrt{2}iE_{\bar{1}1}\lambda^{\bar{1}}_{+} - iE_{\bar{1}2}\lambda^{1}_{+} + \frac{1}{3}(-2iG_{-}\lambda^{\bar{1}}_{-} + \sqrt{2}G_{2}\lambda^{\bar{1}}_{+} - G_{\bar{1}}\lambda^{1}_{+}) = 0\,, \quad \text{(A.18)}$$

$$\sqrt{2}E_{\bar{1}-}\lambda^{\bar{1}}_{-} + \sqrt{2}iE_{\bar{1}\bar{1}}\lambda^{1}_{+} + iE_{\bar{1}2}\lambda^{\bar{1}}_{+} - G_{\bar{1}}\lambda^{\bar{1}}_{+} = 0\,, \quad \text{(A.19)}$$

$$\sqrt{2}E_{\bar{1}+}\lambda^{1}_{+} - \sqrt{2}iE_{\bar{1}1}\lambda^{\bar{1}}_{-} + iE_{\bar{1}2}\lambda^{1}_{-} + \frac{1}{3}(2iG_{+}\lambda^{\bar{1}}_{+} + \sqrt{2}G_{2}\lambda^{\bar{1}}_{-} - G_{\bar{1}}\lambda^{1}_{-}) = 0\,, \quad \text{(A.20)}$$

$$\sqrt{2}E_{\bar{1}+}\lambda^{\bar{1}}_{+} - \sqrt{2}iE_{\bar{1}\bar{1}}\lambda^{1}_{-} - iE_{\bar{1}2}\lambda^{\bar{1}}_{-} - G_{\bar{1}}\lambda^{\bar{1}}_{-} = 0\,. \quad \text{(A.21)}$$

Finally for $\alpha = 2$ we have

$$\sqrt{2}E_{2-}\lambda^{1}_{-} + \sqrt{2}iE_{21}\lambda^{\bar{1}}_{+} - iE_{22}\lambda^{1}_{+} + \frac{1}{3}(\sqrt{2}iG_{-}\lambda^{1}_{-} - \sqrt{2}G_{1}\lambda^{\bar{1}}_{+}) - \frac{2}{3}G_{2}\lambda^{1}_{+} = 0\,, \quad \text{(A.22)}$$

$$\sqrt{2}E_{2-}\lambda^{\bar{1}}_{-} + \sqrt{2}iE_{2\bar{1}}\lambda^{1}_{+} + iE_{22}\lambda^{\bar{1}}_{+} + \frac{1}{3}(-\sqrt{2}iG_{-}\lambda^{\bar{1}}_{-} + \sqrt{2}G_{\bar{1}}\lambda^{1}_{+}) - \frac{2}{3}G_{2}\lambda^{\bar{1}}_{+} = 0\,, \quad \text{(A.23)}$$

$$\sqrt{2}E_{2+}\lambda^{1}_{+} - \sqrt{2}iE_{21}\lambda^{\bar{1}}_{-} + iE_{22}\lambda^{1}_{-} + \frac{1}{3}(\sqrt{2}iG_{+}\lambda^{1}_{+} - \sqrt{2}G_{1}\lambda^{\bar{1}}_{-}) - \frac{2}{3}G_{2}\lambda^{1}_{-} = 0\,, \quad \text{(A.24)}$$

$$\sqrt{2}E_{2+}\lambda^{\bar{1}}_{+} - \sqrt{2}iE_{2\bar{1}}\lambda^{1}_{-} - iE_{22}\lambda^{\bar{1}}_{-} + \frac{1}{3}(-\sqrt{2}iG_{+}\lambda^{\bar{1}}_{+} + \sqrt{2}G_{\bar{1}}\lambda^{1}_{-}) - \frac{2}{3}G_{2}\lambda^{\bar{1}}_{-} = 0\,. \quad \text{(A.25)}$$

Acting on the first Killing spinor $\epsilon = \psi^{1}_{+}$, we find the following constraints

$$E_{++} = E_{+2} = E_{+1} = E_{-+} = E_{-1} = E_{-2} = 0\,, \quad \text{(A.26)}$$

and

$$E_{1+} = E_{11} = E_{12} = E_{1\bar{1}} = 0\,, \quad \text{(A.27)}$$

as well as

$$E_{2+} = E_{21} = E_{22} = 0\,, \quad \text{(A.28)}$$

together with

$$G_{+} = G_{-} = G_{2} = G_{1} = 0\,. \quad \text{(A.29)}$$

We can then substitute these back, finding the following non-vanishing constraints for $\alpha = +$

$$E_{+-}\lambda^{\bar{1}}_{-} = E_{+-}\lambda^{1}_{-} = 0\,, \quad \text{(A.30)}$$

JHEP10(2008)103

for $\alpha = -$

$$E_{--}\lambda^1_- = E_{--}\lambda^{\bar{1}}_- = 0\,, \tag{A.31}$$

for $\alpha = 1$

$$E_{1-}\lambda^1_- = E_{1-}\lambda^{\bar{1}}_- = 0\,, \tag{A.32}$$

for $\alpha = 2$

$$E_{2-}\lambda^1_- = E_{2-}\lambda^{\bar{1}}_- = 0\,. \tag{A.33}$$

We recall from section 4 that the residual gauge transformations preserving $\epsilon = \psi^1_+$ allowed us to place our second Killing spinor $\eta = \lambda^1_+\psi^1_+ + \lambda^{\bar{1}}_+\psi^{\bar{1}}_+ + \lambda^1_-\psi^1_- + \lambda^{\bar{1}}_-\psi^{\bar{1}}_-$ into a form where either $\lambda^\alpha_- = 0$, or $\lambda^\alpha_+ = 0$, for $\alpha = 1, \bar{1}$. In section 6 solutions with $\lambda^\alpha_- = 0$ were found to be only $\frac{1}{4}$ supersymmetric. If we then examine the case $\lambda^\alpha_+ = 0$, we see that we must have $G = X^I G_I = 0$ and $E = 0$.

Evaluating (A.2) for a general Dirac spinor ϵ, yields, for the $\psi^1_+, \psi^{\bar{1}}_+, \psi^1_-, \psi^{\bar{1}}_-$ components

$$S_I\lambda^1_+ - \frac{2}{3}(\sqrt{2}G_{I-}\lambda^1_- + \sqrt{2}iG_{I1}\lambda^{\bar{1}}_+ - iG_{I2}\lambda^1_+) = 0\,, \tag{A.34}$$

$$S_I\lambda^{\bar{1}}_+ - \frac{2}{3}(\sqrt{2}G_{I-}\lambda^{\bar{1}}_- + \sqrt{2}iG_{I\bar{1}}\lambda^1_+ + iG_{I2}\lambda^{\bar{1}}_+) = 0\,, \tag{A.35}$$

$$S_I\lambda^1_- - \frac{2}{3}(\sqrt{2}G_{I+}\lambda^1_+ - \sqrt{2}iG_{I1}\lambda^{\bar{1}}_- + iG_{I2}\lambda^1_-) = 0\,, \tag{A.36}$$

$$S_I\lambda^{\bar{1}}_- - \frac{2}{3}(\sqrt{2}G_{I+}\lambda^{\bar{1}}_+ - \sqrt{2}iG_{I\bar{1}}\lambda^1_- - iG_{I2}\lambda^{\bar{1}}_-) = 0\,, \tag{A.37}$$

where we have used $G = X^I G_I = 0$. Next, we restrict to the case $\epsilon = \psi^1_+$

$$S_I = 0\,, \tag{A.38}$$

$$G_{I2} = 0\,, \tag{A.39}$$

$$G_{I\bar{1}} = 0\,, \tag{A.40}$$

$$G_{I+} = 0\,. \tag{A.41}$$

Substituting back, we find that

$$G_{I-}\lambda^1_- = G_{I-}\lambda^{\bar{1}}_- = 0\,, \tag{A.42}$$

so $G_I = 0$ and $S_I = 0$.

B. The linear system

In this appendix we present the decomposition of the Killing spinor equations acting on a generic Killing spinor (written in an adapted null basis), and then present a special case.

B.1 Solutions with $\epsilon = \lambda^\alpha_+\psi^\alpha_+ + \lambda^\alpha_-\psi^\alpha_-$

The action of the dilatino equations on ϵ is:

$$4i\chi(X^I V_J X^J - \frac{3}{2}Q^{IJ}V_J)\lambda^1_+ + 2\sqrt{2}\partial_- X^I\lambda^1_- + 2\sqrt{2}i\partial_1 X^I\lambda^{\bar{1}}_+ - 2i\partial_2 X^I\lambda^1_+$$

$$+2(F^I_{+-}-X^IH_{+-})\lambda^1_+ + 4i(F^I_{-1}-X^IH_{-1})\lambda^{\bar 1}_- - 2\sqrt{2}i(F^I_{-2}-X^IH_{-2})\lambda^1_-$$
$$+2\sqrt{2}(F^I_{12}-X^IH_{12})\lambda^{\bar 1}_+ + 2(F^I_{1\bar 1}-X^IH_{1\bar 1})\lambda^1_+ = 0\,, \quad \text{(B.1)}$$

$$4i\chi(X^IV_JX^J-\frac{3}{2}Q^{IJ}V_J)\lambda^{\bar 1}_+ + 2\sqrt{2}\partial_-X^I\lambda^{\bar 1}_- + 2\sqrt{2}i\partial_{\bar 1}X^I\lambda^1_+ + 2i\partial_2X^I\lambda^{\bar 1}_+$$
$$+2(F^I_{+-}-X^IH_{+-})\lambda^{\bar 1}_+ + 4i(F^I_{-\bar 1}-X^IH_{-\bar 1})\lambda^1_- + 2\sqrt{2}i(F^I_{-2}-X^IH_{-2})\lambda^{\bar 1}_-$$
$$-2\sqrt{2}(F^I_{\bar 12}-X^IH_{\bar 12})\lambda^1_+ - 2(F^I_{1\bar 1}-X^IH_{1\bar 1})\lambda^{\bar 1}_+ = 0\,, \quad \text{(B.2)}$$

$$4i\chi(X^IV_JX^J-\frac{3}{2}Q^{IJ}V_J)\lambda^1_- + 2\sqrt{2}\partial_+X^I\lambda^1_+ - 2\sqrt{2}i\partial_1X^I\lambda^{\bar 1}_- + 2i\partial_2X^I\lambda^1_-$$
$$-2(F^I_{+-}-X^IH_{+-})\lambda^1_- - 4i(F^I_{+1}-X^IH_{+1})\lambda^{\bar 1}_+ + 2\sqrt{2}i(F^I_{+2}-X^IH_{+2})\lambda^1_+$$
$$+2\sqrt{2}(F^I_{12}-X^IH_{12})\lambda^{\bar 1}_- + 2(F^I_{1\bar 1}-X^IH_{1\bar 1})\lambda^1_- = 0\,, \quad \text{(B.3)}$$

$$4i\chi(X^IV_JX^J-\frac{3}{2}Q^{IJ}V_J)\lambda^{\bar 1}_- + 2\sqrt{2}\partial_+X^I\lambda^{\bar 1}_+ - 2\sqrt{2}i\partial_{\bar 1}X^I\lambda^1_- - 2i\partial_2X^I\lambda^{\bar 1}_-$$
$$-2(F^I_{+-}-X^IH_{+-})\lambda^{\bar 1}_- - 4i(F^I_{+\bar 1}-X^IH_{+\bar 1})\lambda^1_+ - 2\sqrt{2}i(F^I_{+2}-X^IH_{+2})\lambda^{\bar 1}_+$$
$$-2\sqrt{2}(F^I_{\bar 12}-X^IH_{\bar 12})\lambda^1_- - 2(F^I_{1\bar 1}-X^IH_{1\bar 1})\lambda^{\bar 1}_- = 0\,. \quad \text{(B.4)}$$

The action of the gravitino equation on ϵ in the + direction is given by (taking the $\psi^1_+, \psi^{\bar 1}_+, \psi^1_-, \psi^{\bar 1}_-$ components in turn):

$$\left(\partial_+ - \frac{3i\chi}{2}A_+\right)\lambda^1_+ - \frac{3}{4}(\sqrt{2}H_{+-}\lambda^1_- + \sqrt{2}iH_{+1}\lambda^{\bar 1}_+ - iH_{+2}\lambda^1_+)$$
$$+\frac{1}{2}(-\omega_{+,+-}\lambda^1_+ - 2i\omega_{+,-1}\lambda^{\bar 1}_- + \sqrt{2}i\omega_{+,-2}\lambda^1_- - \sqrt{2}\omega_{+,12}\lambda^{\bar 1}_+ - \omega_{+,1\bar 1}\lambda^1_+)$$
$$+\frac{\sqrt{2}}{4}(H_{+-}\lambda^1_- + 2iH_{+1}\lambda^{\bar 1}_+ - \sqrt{2}iH_{+2}\lambda^1_+ - \sqrt{2}H_{12}\lambda^{\bar 1}_- - H_{1\bar 1}\lambda^1_-)$$
$$+\frac{i\chi}{\sqrt{2}}V_IX^I\lambda^1_- = 0\,, \quad \text{(B.5)}$$

$$\left(\partial_+ - \frac{3i\chi}{2}A_+\right)\lambda^{\bar 1}_+ - \frac{3}{4}(\sqrt{2}H_{+-}\lambda^{\bar 1}_- + \sqrt{2}iH_{+\bar 1}\lambda^1_+ + iH_{+2}\lambda^{\bar 1}_+)$$
$$+\frac{1}{2}(-\omega_{+,+-}\lambda^{\bar 1}_+ - 2i\omega_{+,-\bar 1}\lambda^1_- - \sqrt{2}i\omega_{+,-2}\lambda^{\bar 1}_- + \sqrt{2}\omega_{+,\bar 12}\lambda^1_+ + \omega_{+,1\bar 1}\lambda^{\bar 1}_+)$$
$$+\frac{\sqrt{2}}{4}(H_{+-}\lambda^{\bar 1}_- + 2iH_{+\bar 1}\lambda^1_+ + \sqrt{2}iH_{+2}\lambda^{\bar 1}_+ + \sqrt{2}H_{\bar 12}\lambda^1_- + H_{1\bar 1}\lambda^{\bar 1}_-)$$
$$+\frac{i\chi}{\sqrt{2}}V_IX^I\lambda^{\bar 1}_- = 0\,, \quad \text{(B.6)}$$

$$\left(\partial_+ - \frac{3i\chi}{2}A_+\right)\lambda^1_- - \frac{3}{4}(-\sqrt{2}iH_{+1}\lambda^{\bar 1}_- + iH_{+2}\lambda^1_-)$$
$$+\frac{1}{2}(\omega_{+,+-}\lambda^1_- + 2i\omega_{+,+1}\lambda^{\bar 1}_+ - \sqrt{2}i\omega_{+,+2}\lambda^1_+ - \sqrt{2}\omega_{+,12}\lambda^{\bar 1}_- - \omega_{+,1\bar 1}\lambda^1_-) = 0\,, \quad \text{(B.7)}$$

$$\left(\partial_+ - \frac{3i\chi}{2}A_+\right)\lambda^{\bar 1}_- - \frac{3}{4}(-\sqrt{2}iH_{+\bar 1}\lambda^1_- - iH_{+2}\lambda^{\bar 1}_-)$$
$$+\frac{1}{2}(\omega_{+,+-}\lambda^{\bar 1}_- + 2i\omega_{+,+\bar 1}\lambda^1_+ + \sqrt{2}i\omega_{+,+2}\lambda^{\bar 1}_+ + \sqrt{2}\omega_{+,\bar 12}\lambda^1_- + \omega_{+,1\bar 1}\lambda^{\bar 1}_-) = 0\,. \quad \text{(B.8)}$$

JHEP10(2008)103

In the − direction

$$\left(\partial_- - \frac{3i\chi}{2}A_-\right)\lambda^1_+ - \frac{3}{4}(\sqrt{2}iH_{-1}\lambda^{\bar{1}}_+ - iH_{-2}\lambda^1_+)$$
$$+\frac{1}{2}(-\omega_{-,+-}\lambda^1_+ - 2i\omega_{-,-1}\lambda^{\bar{1}}_- + \sqrt{2}i\omega_{-,-2}\lambda^1_- - \sqrt{2}\omega_{-,12}\lambda^{\bar{1}}_+ - \omega_{-,1\bar{1}}\lambda^1_+) = 0\,, \quad \text{(B.9)}$$

$$\left(\partial_- - \frac{3i\chi}{2}A_-\right)\lambda^{\bar{1}}_+ - \frac{3}{4}(\sqrt{2}iH_{-\bar{1}}\lambda^1_+ + iH_{-2}\lambda^{\bar{1}}_+)$$
$$+\frac{1}{2}(-\omega_{-,+-}\lambda^{\bar{1}}_+ - 2i\omega_{-,-\bar{1}}\lambda^1_- - \sqrt{2}i\omega_{-,-2}\lambda^{\bar{1}}_- + \sqrt{2}\omega_{-,\bar{1}2}\lambda^1_+ + \omega_{-,1\bar{1}}\lambda^{\bar{1}}_+) = 0\,, \quad \text{(B.10)}$$

$$\left(\partial_- - \frac{3i\chi}{2}A_-\right)\lambda^1_- - \frac{3}{4}(-\sqrt{2}H_{+-}\lambda^1_+ - \sqrt{2}iH_{-1}\lambda^{\bar{1}}_- + iH_{-2}\lambda^1_-)$$
$$+\frac{1}{2}(\omega_{-,+-}\lambda^1_- + 2i\omega_{-,+1}\lambda^{\bar{1}}_+ - \sqrt{2}i\omega_{-,+2}\lambda^1_+ - \sqrt{2}\omega_{-,12}\lambda^{\bar{1}}_- - \omega_{-,1\bar{1}}\lambda^1_-)$$
$$+\frac{\sqrt{2}}{4}(-H_{+-}\lambda^1_+ - 2iH_{-1}\lambda^{\bar{1}}_- + \sqrt{2}iH_{-2}\lambda^1_- - \sqrt{2}H_{12}\lambda^{\bar{1}}_+ - H_{1\bar{1}}\lambda^1_+)$$
$$+\frac{i\chi}{\sqrt{2}}V_I X^I\lambda^1_+ = 0\,, \quad \text{(B.11)}$$

$$\left(\partial_- - \frac{3i\chi}{2}A_-\right)\lambda^{\bar{1}}_- - \frac{3}{4}(-\sqrt{2}H_{+-}\lambda^{\bar{1}}_+ - \sqrt{2}iH_{-\bar{1}}\lambda^1_- - iH_{-2}\lambda^{\bar{1}}_-)$$
$$+\frac{1}{2}(\omega_{-,+-}\lambda^{\bar{1}}_- + 2i\omega_{-,+\bar{1}}\lambda^1_+ + \sqrt{2}i\omega_{-,+2}\lambda^{\bar{1}}_+ + \sqrt{2}\omega_{-,\bar{1}2}\lambda^1_- + \omega_{-,1\bar{1}}\lambda^{\bar{1}}_-)$$
$$+\frac{\sqrt{2}}{4}(-H_{+-}\lambda^{\bar{1}}_+ - 2iH_{-\bar{1}}\lambda^1_- - \sqrt{2}iH_{-2}\lambda^{\bar{1}}_- + \sqrt{2}H_{\bar{1}2}\lambda^1_+ + H_{1\bar{1}}\lambda^{\bar{1}}_+)$$
$$+\frac{i\chi}{\sqrt{2}}V_I X^I\lambda^{\bar{1}}_+ = 0\,. \quad \text{(B.12)}$$

In the 1 direction

$$\left(\partial_1 - \frac{3i\chi}{2}A_1\right)\lambda^1_+ - \frac{3}{4}(-\sqrt{2}H_{-1}\lambda^1_- - iH_{12}\lambda^1_+)$$
$$+\frac{1}{2}(-\omega_{1,+-}\lambda^1_+ - 2i\omega_{1,-1}\lambda^{\bar{1}}_- + \sqrt{2}i\omega_{1,-2}\lambda^1_- - \sqrt{2}\omega_{1,12}\lambda^{\bar{1}}_+ - \omega_{1,1\bar{1}}\lambda^1_+) = 0\,, \quad \text{(B.13)}$$

$$\left(\partial_1 - \frac{3i\chi}{2}A_1\right)\lambda^{\bar{1}}_+ - \frac{3}{4}(-\sqrt{2}H_{-1}\lambda^{\bar{1}}_- + \sqrt{2}iH_{1\bar{1}}\lambda^1_+ + iH_{12}\lambda^{\bar{1}}_+)$$
$$+\frac{1}{2}(-\omega_{1,+-}\lambda^{\bar{1}}_+ - 2i\omega_{1,-\bar{1}}\lambda^1_- - \sqrt{2}i\omega_{1,-2}\lambda^{\bar{1}}_- + \sqrt{2}\omega_{1,\bar{1}2}\lambda^1_+ + \omega_{1,1\bar{1}}\lambda^{\bar{1}}_+)$$
$$-\frac{i\sqrt{2}}{4}(-H_{+-}\lambda^1_+ - 2iH_{-1}\lambda^{\bar{1}}_- + \sqrt{2}iH_{-2}\lambda^1_- - \sqrt{2}H_{12}\lambda^{\bar{1}}_+ - H_{1\bar{1}}\lambda^1_+)$$
$$+\frac{\chi}{\sqrt{2}}V_I X^I\lambda^1_+ = 0\,, \quad \text{(B.14)}$$

$$\left(\partial_1 - \frac{3i\chi}{2}A_1\right)\lambda^1_- - \frac{3}{4}(-\sqrt{2}H_{+1}\lambda^1_+ + iH_{12}\lambda^1_-)$$
$$+\frac{1}{2}(\omega_{1,+-}\lambda^1_- + 2i\omega_{1,+1}\lambda^{\bar{1}}_+ - \sqrt{2}i\omega_{1,+2}\lambda^1_+ - \sqrt{2}\omega_{1,12}\lambda^{\bar{1}}_- - \omega_{1,1\bar{1}}\lambda^1_-) = 0\,, \quad \text{(B.15)}$$

$$\left(\partial_1 - \frac{3i\chi}{2}A_1\right)\lambda^{\bar{1}}_- - \frac{3}{4}(-\sqrt{2}H_{+1}\lambda^{\bar{1}}_+ - \sqrt{2}iH_{1\bar{1}}\lambda^1_- + iH_1{}^2\lambda^{\bar{1}}_-)$$

JHEP10(2008)103

$$+\frac{1}{2}(\omega_{1,+-}\lambda^{\bar{1}}_{-}+2i\omega_{1,+\bar{1}}\lambda^{1}_{+}+\sqrt{2}i\omega_{1,+2}\lambda^{\bar{1}}_{+}+\sqrt{2}\omega_{1,\bar{1}2}\lambda^{1}_{-}+\omega_{1,1\bar{1}}\lambda^{\bar{1}}_{-})$$
$$+\frac{\sqrt{2}i}{4}(H_{+-}\lambda^{1}_{-}+2iH_{+1}\lambda^{\bar{1}}_{+}-\sqrt{2}iH_{+2}\lambda^{1}_{+}-\sqrt{2}H_{12}\lambda^{\bar{1}}_{-}-H_{1\bar{1}}\lambda^{1}_{-})$$
$$-\frac{\chi}{\sqrt{2}}V_I X^I\lambda^{1}_{-}=0\,. \quad \text{(B.16)}$$

In the $\bar{1}$ direction

$$\Big(\partial_{\bar{1}}-\frac{3i\chi}{2}A_{\bar{1}}\Big)\lambda^{1}_{+}-\frac{3}{4}(-\sqrt{2}H_{-\bar{1}}\lambda^{1}_{-}-\sqrt{2}iH_{1\bar{1}}\lambda^{\bar{1}}_{+}-iH_{\bar{1}2}\lambda^{1}_{+})$$
$$+\frac{1}{2}(-\omega_{\bar{1},+-}\lambda^{1}_{+}-2i\omega_{\bar{1},-1}\lambda^{\bar{1}}_{-}+\sqrt{2}i\omega_{\bar{1},-2}\lambda^{1}_{-}-\sqrt{2}\omega_{\bar{1},12}\lambda^{\bar{1}}_{+}-\omega_{\bar{1},1\bar{1}}\lambda^{1}_{+})$$
$$-\frac{\sqrt{2}i}{4}(-H_{+-}\lambda^{\bar{1}}_{+}-2iH_{-\bar{1}}\lambda^{1}_{-}-\sqrt{2}iH_{-2}\lambda^{\bar{1}}_{-}+\sqrt{2}H_{\bar{1}2}\lambda^{1}_{+}+H_{1\bar{1}}\lambda^{\bar{1}}_{+})$$
$$+\frac{\chi}{\sqrt{2}}V_I X^I\lambda^{\bar{1}}_{+}=0\,, \quad \text{(B.17)}$$
$$\Big(\partial_{\bar{1}}-\frac{3i\chi}{2}A_{\bar{1}}\Big)\lambda^{\bar{1}}_{+}-\frac{3}{4}(-\sqrt{2}H_{-\bar{1}}\lambda^{\bar{1}}_{-}+iH_{\bar{1}2}\lambda^{\bar{1}}_{+})$$
$$+\frac{1}{2}(-\omega_{\bar{1},+-}\lambda^{\bar{1}}_{+}-2i\omega_{\bar{1},-\bar{1}}\lambda^{1}_{-}-\sqrt{2}i\omega_{\bar{1},-2}\lambda^{\bar{1}}_{-}+\sqrt{2}\omega_{\bar{1},\bar{1}2}\lambda^{1}_{+}+\omega_{\bar{1},1\bar{1}}\lambda^{\bar{1}}_{+})=0\,, \quad \text{(B.18)}$$
$$\Big(\partial_{\bar{1}}-\frac{3i\chi}{2}A_{\bar{1}}\Big)\lambda^{1}_{-}-\frac{3}{4}(-\sqrt{2}H_{+\bar{1}}\lambda^{1}_{+}+\sqrt{2}iH_{1\bar{1}}\lambda^{\bar{1}}_{-}+iH_{\bar{1}2}\lambda^{1}_{-})$$
$$+\frac{1}{2}(\omega_{\bar{1},+-}\lambda^{1}_{-}+2i\omega_{\bar{1},+1}\lambda^{\bar{1}}_{+}-\sqrt{2}i\omega_{\bar{1},+2}\lambda^{1}_{+}-\sqrt{2}\omega_{\bar{1},12}\lambda^{\bar{1}}_{-}-\omega_{\bar{1},1\bar{1}}\lambda^{1}_{-})$$
$$+\frac{\sqrt{2}i}{4}(H_{+-}\lambda^{\bar{1}}_{-}+2iH_{+\bar{1}}\lambda^{1}_{+}+\sqrt{2}iH_{+2}\lambda^{\bar{1}}_{+}+\sqrt{2}H_{\bar{1}2}\lambda^{1}_{-}+H_{1\bar{1}}\lambda^{\bar{1}}_{-})$$
$$-\frac{\chi}{\sqrt{2}}V_I X^I\lambda^{\bar{1}}_{-}=0\,, \quad \text{(B.19)}$$
$$\Big(\partial_{\bar{1}}-\frac{3i\chi}{2}A_{\bar{1}}\Big)\lambda^{\bar{1}}_{-}-\frac{3}{4}(-\sqrt{2}H_{+\bar{1}}\lambda^{\bar{1}}_{+}-iH_{\bar{1}2}\lambda^{\bar{1}}_{-})$$
$$+\frac{1}{2}(\omega_{\bar{1},+-}\lambda^{\bar{1}}_{-}+2i\omega_{\bar{1},+\bar{1}}\lambda^{1}_{+}+\sqrt{2}i\omega_{\bar{1},+2}\lambda^{\bar{1}}_{+}+\sqrt{2}\omega_{\bar{1},\bar{1}2}\lambda^{1}_{-}+\omega_{\bar{1},1\bar{1}}\lambda^{\bar{1}}_{-})=0\,. \quad \text{(B.20)}$$

Finally, in the 2 direction

$$\Big(\partial_{2}-\frac{3i\chi}{2}A_{2}\Big)\lambda^{1}_{+}-\frac{3}{4}(-\sqrt{2}H_{-2}\lambda^{1}_{-}-\sqrt{2}iH_{12}\lambda^{\bar{1}}_{+})$$
$$+\frac{1}{2}(-\omega_{2,+-}\lambda^{1}_{+}-2i\omega_{2,-1}\lambda^{\bar{1}}_{-}+\sqrt{2}i\omega_{2,-2}\lambda^{1}_{-}-\sqrt{2}\omega_{2,12}\lambda^{\bar{1}}_{+}-\omega_{2,1\bar{1}}\lambda^{1}_{+})$$
$$+\frac{i}{4}(-H_{+-}\lambda^{1}_{+}-2iH_{-1}\lambda^{\bar{1}}_{-}+\sqrt{2}iH_{-2}\lambda^{1}_{-}-\sqrt{2}H_{12}\lambda^{\bar{1}}_{+}-H_{1\bar{1}}\lambda^{1}_{+})$$
$$-\frac{\chi}{2}V_I X^I\lambda^{1}_{+}=0\,, \quad \text{(B.21)}$$
$$\Big(\partial_{2}-\frac{3i\chi}{2}A_{2}\Big)\lambda^{\bar{1}}_{+}-\frac{3}{4}(-\sqrt{2}H_{-2}\lambda^{\bar{1}}_{-}-\sqrt{2}iH_{\bar{1}2}\lambda^{1}_{+})$$
$$+\frac{1}{2}(-\omega_{2,+-}\lambda^{\bar{1}}_{+}-2i\omega_{2,-\bar{1}}\lambda^{1}_{-}-\sqrt{2}i\omega_{2,-2}\lambda^{\bar{1}}_{-}+\sqrt{2}\omega_{2,\bar{1}2}\lambda^{1}_{+}+\omega_{2,1\bar{1}}\lambda^{\bar{1}}_{+})$$

$$-\frac{i}{4}(-H_{+-}\lambda^{\bar{1}}_{+}-2iH_{-\bar{1}}\lambda^{1}_{-}-\sqrt{2}iH_{-2}\lambda^{\bar{1}}_{-}+\sqrt{2}H_{\bar{1}2}\lambda^{1}_{+}+H_{1\bar{1}}\lambda^{\bar{1}}_{+})$$
$$+\frac{\chi}{2}V_I X^I\lambda^{\bar{1}}_{+}=0\,, \tag{B.22}$$

$$\left(\partial_2-\frac{3i\chi}{2}A_2\right)\lambda^{1}_{-}-\frac{3}{4}(-\sqrt{2}H_{+2}\lambda^{1}_{+}+\sqrt{2}iH_{12}\lambda^{\bar{1}}_{-})$$
$$+\frac{1}{2}(\omega_{2,+-}\lambda^{1}_{-}+2i\omega_{2,+1}\lambda^{\bar{1}}_{+}-\sqrt{2}i\omega_{2,+2}\lambda^{1}_{+}-\sqrt{2}\omega_{2,12}\lambda^{\bar{1}}_{-}-\omega_{2,1\bar{1}}\lambda^{1}_{-})$$
$$-\frac{i}{4}(H_{+-}\lambda^{1}_{-}+2iH_{+1}\lambda^{\bar{1}}_{+}-\sqrt{2}iH_{+2}\lambda^{1}_{+}-\sqrt{2}H_{12}\lambda^{\bar{1}}_{-}-H_{1\bar{1}}\lambda^{1}_{-})+\frac{\chi}{2}V_I X^I\lambda^{1}_{-}=0\,, \tag{B.23}$$

$$\left(\partial_2-\frac{3i\chi}{2}A_2\right)\lambda^{\bar{1}}_{-}-\frac{3}{4}(-\sqrt{2}H_{+2}\lambda^{\bar{1}}_{+}+\sqrt{2}iH_{\bar{1}2}\lambda^{1}_{-})$$
$$+\frac{1}{2}(\omega_{2,+-}\lambda^{\bar{1}}_{-}+2i\omega_{2,+\bar{1}}\lambda^{1}_{+}+\sqrt{2}i\omega_{2,+2}\lambda^{\bar{1}}_{+}+\sqrt{2}\omega_{2,\bar{1}2}\lambda^{1}_{-}+\omega_{2,1\bar{1}}\lambda^{\bar{1}}_{-})$$
$$+\frac{i}{4}(H_{+-}\lambda^{\bar{1}}_{-}+2iH_{+\bar{1}}\lambda^{1}_{+}+\sqrt{2}iH_{+2}\lambda^{\bar{1}}_{+}+\sqrt{2}H_{\bar{1}2}\lambda^{1}_{-}+H_{1\bar{1}}\lambda^{\bar{1}}_{-})-\frac{\chi}{2}V_I X^I\lambda^{\bar{1}}_{-}=0\,. \tag{B.24}$$

B.2 Constraints on half-supersymmetric solutions

Substituting the constraints obtained in section 4, for quarter-supersymmetric solutions with $\epsilon=\psi^1_+$, back into the dilatino equations we find

$$2\sqrt{2}\partial_- X^I\lambda^{1}_{-}+4i(F^I_{-1}-X^I H_{-1})\lambda^{\bar{1}}_{-}-2\sqrt{2}i(F^I_{-2}-X^I H_{-2})\lambda^{1}_{-}=0\,, \tag{B.25}$$

$$8i\chi\left(X^I V_J X^J-\frac{3}{2}Q^{IJ}V_J\right)\lambda^{\bar{1}}_{+}+2\sqrt{2}\partial_- X^I\lambda^{\bar{1}}_{-}$$
$$+4i(F^I_{-\bar{1}}-X^I H_{-\bar{1}})\lambda^{1}_{-}+2\sqrt{2}i(F^I_{-2}-X^I H_{-2})\lambda^{\bar{1}}_{-}=0\,, \tag{B.26}$$

$$4\sqrt{2}i\partial_1 X^I\lambda^{\bar{1}}_{-}-4i\partial_2 X^I\lambda^{1}_{-}=0\,, \tag{B.27}$$

$$4\sqrt{2}i\partial_{\bar{1}} X^I\lambda^{1}_{-}+4i\partial_2 X^I\lambda^{\bar{1}}_{-}-8i\chi\left(X^I V_J X^J-\frac{3}{2}Q^{IJ}V_J\right)\lambda^{\bar{1}}_{-}=0\,. \tag{B.28}$$

Substituting the constraints back into the gravitino equations yields, in the + direction:

$$\partial_+\lambda^{1}_{+}-i\omega_{+,-1}\lambda^{\bar{1}}_{-}+\frac{\sqrt{2}i}{2}\omega_{+,-2}\lambda^{1}_{-}+\frac{i}{2}\omega_{2,12}\lambda^{\bar{1}}_{-}+\frac{\sqrt{2}i}{2}\omega_{-,+2}\lambda^{1}_{-}=0\,, \tag{B.29}$$

$$\partial_+\lambda^{\bar{1}}_{+}+\omega_{+,1\bar{1}}\lambda^{\bar{1}}_{+}-i\omega_{+,-\bar{1}}\lambda^{1}_{-}-\frac{\sqrt{2}i}{2}\omega_{+,-2}\lambda^{\bar{1}}_{-}$$
$$+\frac{i}{2}\omega_{2,\bar{1}2}\lambda^{1}_{-}+\frac{\sqrt{2}i}{6}\omega_{-,+2}\lambda^{\bar{1}}_{-}-\frac{\sqrt{2}i}{3}\omega_{1,\bar{1}2}\lambda^{\bar{1}}_{-}=0\,, \tag{B.30}$$

$$\partial_+\lambda^{1}_{-}=0\,, \tag{B.31}$$

$$(\partial_++\omega_{+,1\bar{1}})\lambda^{\bar{1}}_{-}=0\,. \tag{B.32}$$

In the − direction:

$$\partial_-\lambda^{1}_{+}-i\omega_{-,-1}\lambda^{\bar{1}}_{-}+\frac{\sqrt{2}i}{2}\omega_{-,-2}\lambda^{1}_{-}=0\,, \tag{B.33}$$

$$(\partial_- - 3i\chi A_-)\lambda^{\bar{1}}_+ - i\omega_{-,-\bar{1}}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{-,-2}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.34)}$$

$$(\partial_- - 2i\chi A_-)\lambda^1_- - \frac{2\sqrt{2}}{3}\omega_{-,12}\lambda^{\bar{1}}_- - \frac{2}{3}\omega_{-,1\bar{1}}\lambda^1_- = 0\,, \quad \text{(B.35)}$$

$$(\partial_- - i\chi A_-)\lambda^{\bar{1}}_- + \frac{2\sqrt{2}}{3}\omega_{-,\bar{1}2}\lambda^1_- + \frac{2}{3}\omega_{-,1\bar{1}}\lambda^{\bar{1}}_- + \frac{\sqrt{2}i}{3}(2\omega_{-,+2} - \omega_{1,\bar{1}2})\lambda^{\bar{1}}_+ = 0\,. \quad \text{(B.36)}$$

In the 1 direction:

$$\partial_1\lambda^1_+ + \frac{\sqrt{2}i}{2}\omega_{-,12}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{1,-2}\lambda^1_- - i\omega_{1,-1}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.37)}$$

$$\partial_1\lambda^{\bar{1}}_+ + \omega_{1,1\bar{1}}\lambda^{\bar{1}}_+ - 2\omega_{1,+-}\lambda^{\bar{1}}_+ - i\omega_{1,-\bar{1}}\lambda^1_- + \chi A_-\lambda^1_- + \frac{\sqrt{2}i}{6}\omega_{-,12}\lambda^{\bar{1}}_- - \frac{i}{3}\omega_{-,1\bar{1}}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{1,-2}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.38)}$$

$$(\partial_1 - 2\omega_{1,+-})\lambda^1_- = 0\,, \quad \text{(B.39)}$$

$$\partial_1\lambda^{\bar{1}}_- + \omega_{1,1\bar{1}}\lambda^{\bar{1}}_- + \sqrt{2}\omega_{1,\bar{1}2}\lambda^1_- = 0\,. \quad \text{(B.40)}$$

In the $\bar{1}$ direction:

$$\partial_{\bar{1}}\lambda^1_+ + \frac{\sqrt{2}}{3}(2\omega_{-,+2} - \omega_{1,\bar{1}2})\lambda^{\bar{1}}_+ - \frac{\sqrt{2}i}{6}\omega_{-,\bar{1}2}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{\bar{1},-2}\lambda^1_- - i\omega_{\bar{1},-1}\lambda^{\bar{1}}_- + \frac{i}{3}\omega_{-,1\bar{1}}\lambda^{\bar{1}}_- - \chi A_-\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.41)}$$

$$\partial_{\bar{1}}\lambda^{\bar{1}}_+ + 2\omega_{\bar{1},+-}\lambda^{\bar{1}}_+ + \omega_{\bar{1},1\bar{1}}\lambda^{\bar{1}}_+ - i\omega_{\bar{1},-\bar{1}}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{\bar{1},-2}\lambda^{\bar{1}}_- - \frac{\sqrt{2}i}{2}\omega_{-,\bar{1}2}\lambda^{\bar{1}}_- = 0, \quad \text{(B.42)}$$

$$\partial_{\bar{1}}\lambda^1_- + 2\omega_{\bar{1},+-}\lambda^1_- - \frac{2\sqrt{2}}{3}(\omega_{-,+2} + \omega_{\bar{1},12})\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.43)}$$

$$(\partial_{\bar{1}} + \omega_{\bar{1},1\bar{1}})\lambda^{\bar{1}}_- = 0\,. \quad \text{(B.44)}$$

In the 2 direction:

$$\partial_2\lambda^1_+ - \sqrt{2}\lambda^1_-\Big(-\chi A_- + \frac{i}{3}\omega_{-,1\bar{1}}\Big) - i\omega_{2,-1}\lambda^{\bar{1}}_- + \frac{\sqrt{2}i}{2}\omega_{2,-2}\lambda^1_- + \frac{i}{3}\omega_{-,12}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.45)}$$

$$\partial_2\lambda^{\bar{1}}_+ + \Big(\frac{2}{3}\omega_{-,+2} - \frac{1}{3}\omega_{1,\bar{1}2} + \omega_{2,1\bar{1}}\Big)\lambda^{\bar{1}}_+ - i\omega_{2,-\bar{1}}\lambda^1_- + \frac{i}{3}\omega_{-,\bar{1}2}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{2,-2}\lambda^{\bar{1}}_- - \sqrt{2}\Big(-\chi A_- + \frac{i}{3}\omega_{-,1\bar{1}}\Big)\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.46)}$$

$$\partial_2\lambda^1_- - \sqrt{2}\omega_{2,12}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.47)}$$

$$\partial_2\lambda^{\bar{1}}_- + \sqrt{2}\omega_{2,\bar{1}2}\lambda^1_- - (\frac{2}{3}\omega_{-,+2} - \frac{1}{3}\omega_{1,\bar{1}2} - \omega_{2,1\bar{1}})\lambda^{\bar{1}}_- = 0\,. \quad \text{(B.48)}$$

B.3 Solutions with $\lambda^\alpha_+ = 0$

In the case where $\lambda^\alpha_+ = 0$ we can reduce the dilatino equations to:

$$2\sqrt{2}\partial_- X^I\lambda^1_- + 4i(F^I_{-1} - X^I H_{-1})\lambda^{\bar{1}}_- - 2\sqrt{2}i(F^I_{-2} - X^I H_{-2})\lambda^1_- = 0\,, \quad \text{(B.49)}$$

$$2\sqrt{2}\partial_- X^I\lambda^{\bar{1}}_- + 4i(F^I_{-\bar{1}} - X^I H_{-\bar{1}})\lambda^1_- + 2\sqrt{2}i(F^I_{-2} - X^I H_{-2})\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.50)}$$

JHEP10(2008)103

$$4\sqrt{2}i\partial_1 X^I\lambda^{\bar{1}}_- - 4i\partial_2 X^I\lambda^1_- = 0\,, \tag{B.51}$$

$$4\sqrt{2}i\partial_{\bar{1}} X^I\lambda^1_- + 4i\partial_2 X^I\lambda^{\bar{1}}_- - 8i\chi\Big(X^I V_J X^J - \frac{3}{2}Q^{IJ}V_J\Big)\lambda^{\bar{1}}_- = 0\,. \tag{B.52}$$

The gravitino equations reduce to, in the + direction:

$$i\omega_{+,-1}\lambda^{\bar{1}}_- - \frac{\sqrt{2}i}{2}\omega_{+,-2}\lambda^1_- - \frac{i}{2}\omega_{2,12}\lambda^{\bar{1}}_- - \frac{\sqrt{2}i}{2}\omega_{-,+2}\lambda^1_- = 0\,, \tag{B.53}$$

$$i\omega_{+,-\bar{1}}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{+,-2}\lambda^{\bar{1}}_- - \frac{i}{2}\omega_{2,\bar{1}2}\lambda^1_- - \frac{\sqrt{2}i}{6}\omega_{-,+2}\lambda^{\bar{1}}_- + \frac{\sqrt{2}i}{3}\omega_{1,\bar{1}2}\lambda^{\bar{1}}_- = 0\,, \tag{B.54}$$

$$\partial_+\lambda^1_- = 0\,, \tag{B.55}$$

$$(\partial_+ + \omega_{+,1\bar{1}})\lambda^{\bar{1}}_- = 0\,. \tag{B.56}$$

In the − direction:

$$i\omega_{-,-1}\lambda^{\bar{1}}_- - \frac{\sqrt{2}i}{2}\omega_{-,-2}\lambda^1_- = 0, \tag{B.57}$$

$$i\omega_{-,-\bar{1}}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{-,-2}\lambda^{\bar{1}}_- = 0\,, \tag{B.58}$$

$$(\partial_- - 2i\chi A_-)\lambda^1_- - \frac{2\sqrt{2}}{3}\omega_{-,12}\lambda^{\bar{1}}_- - \frac{2}{3}\omega_{-,1\bar{1}}\lambda^1_- = 0\,, \tag{B.59}$$

$$(\partial_- - i\chi A_-)\lambda^{\bar{1}}_- + \frac{2\sqrt{2}}{3}\omega_{-,\bar{1}2}\lambda^1_- + \frac{2}{3}\omega_{-,1\bar{1}}\lambda^{\bar{1}}_- = 0\,. \tag{B.60}$$

In the 1 direction:

$$\frac{\sqrt{2}i}{2}\omega_{-,12}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{1,-2}\lambda^1_- - i\omega_{1,-1}\lambda^{\bar{1}}_- = 0\,, \tag{B.61}$$

$$i\omega_{1,-\bar{1}}\lambda^1_- - \chi A_-\lambda^1_- - \frac{\sqrt{2}i}{6}\omega_{-,12}\lambda^{\bar{1}}_- + \frac{i}{3}\omega_{-,1\bar{1}}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{1,-2}\lambda^{\bar{1}}_- = 0\,, \tag{B.62}$$

$$(\partial_1 - 2\omega_{1,+-})\lambda^1_- = 0\,, \tag{B.63}$$

$$\partial_1\lambda^{\bar{1}}_- + \omega_{1,1\bar{1}}\lambda^{\bar{1}}_- + \sqrt{2}\omega_{1,\bar{1}2}\lambda^1_- = 0\,. \tag{B.64}$$

In the $\bar{1}$ direction:

$$\frac{\sqrt{2}i}{6}\omega_{-,\bar{1}2}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{\bar{1},-2}\lambda^1_- + i\omega_{\bar{1},-1}\lambda^{\bar{1}}_- - \frac{i}{3}\omega_{-,1\bar{1}}\lambda^{\bar{1}}_- + \chi A_-\lambda^{\bar{1}}_- = 0\,, \tag{B.65}$$

$$i\omega_{\bar{1},-\bar{1}}\lambda^1_- + \frac{\sqrt{2}i}{2}\omega_{\bar{1},-2}\lambda^{\bar{1}}_- + \frac{\sqrt{2}i}{2}\omega_{-,\bar{1}2}\lambda^{\bar{1}}_- = 0\,, \tag{B.66}$$

$$\partial_{\bar{1}}\lambda^1_- + 2\omega_{\bar{1},+-}\lambda^1_- - \frac{2\sqrt{2}}{3}(\omega_{-,+2} + \omega_{\bar{1},12})\lambda^{\bar{1}}_- = 0\,, \tag{B.67}$$

$$(\partial_{\bar{1}} + \omega_{\bar{1},1\bar{1}})\lambda^{\bar{1}}_- = 0\,. \tag{B.68}$$

In the 2 direction:

$$\sqrt{2}\lambda^1_-\Big(-\chi A_- - \frac{i}{3}\omega_{-,1\bar{1}}\Big) + i\omega_{2,-1}\lambda^{\bar{1}}_- - \frac{\sqrt{2}i}{2}\omega_{2,-2}\lambda^1_- - \frac{i}{3}\omega_{-,12}\lambda^{\bar{1}}_- = 0\,, \tag{B.69}$$

JHEP10(2008)103

$$i\omega_{2,-\bar{1}}\lambda^1_- + \frac{i}{3}\omega_{-,\bar{1}2}\lambda^1_- - \frac{\sqrt{2}i}{2}\omega_{2,-2}\lambda^{\bar{1}}_- - \sqrt{2}\Big(- \chi A_- + \frac{i}{3}\omega_{-,1\bar{1}}\Big)\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.70)}$$

$$\partial_2\lambda^1_- - \sqrt{2}\omega_{2,12}\lambda^{\bar{1}}_- = 0\,, \quad \text{(B.71)}$$

$$\partial_2\lambda^{\bar{1}}_- + \sqrt{2}\omega_{2,\bar{1}2}\lambda^1_- - \Big(\frac{2}{3}\omega_{-,+2} - \frac{1}{3}\omega_{1,\bar{1}2} - \omega_{2,1\bar{1}}\Big)\lambda^{\bar{1}}_- = 0\,. \quad \text{(B.72)}$$

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JHEP10(2008)103